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Analytical Study on the Dynamic Characteristics of Cable Networks

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Analytical Study on the Dynamic Characteristics of Cable Networks

By

Javaid Ahmad

A Thesis

Submitted to the Faculty of Graduate Studies
through the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for
the Degree of Master of Applied Science at the
University of Windsor

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2012

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Analytical Study on the Dynamic Characteristics of Cable Networks

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DECLARATION OF CO-AUTHORSHIP/PREVIOUS PUBLICATIONS

This thesis includes 3 original manuscripts, 2 already submitted and 1 soon to be submitted to peer reviewed journals. In addition, a conference paper has already been published based on part of the work presented in Chapters 2 and 3. They are listed as follows:

Thesis Chapter	Publication Title	Publication Status
Chapters 2 and 3 (selected parts)	Javaid Ahmad, Shaohong Cheng, 2011. Analytical Study on In-plane Modal Properties of a General Cable Network. The 9 th International Symposium on Cable Dynamics, Shanghai, China, pp. 32-39.	Published
Chapter 2	Javaid Ahmad, Shaohong Cheng, 2012. Analytical Study on In-plane Modal Behaviour of Stay Cables Connected by Cross-ties: Part I: Basic Cable Network, Journal of Sound and Vibration	Submitted
Chapter 3	Javaid Ahmad, Shaohong Cheng, 2012. Analytical Study on In-plane Modal Behaviour of Stay Cables Connected by Cross-ties: Part II: General Cable Network, Journal of Sound and Vibration	Submitted
Chapter 4	Javaid Ahmad, Shaohong Cheng, 2012. In-plane Free Vibration of a Two-cable Network with Flexible Cross-tie , Journal of Structural Engineering, ASCE	To be submitted

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ABSTRACT

Bridge stay cables are vulnerable to dynamic excitations by wind. There are different countermeasures to suppress unfavorable bridge stay cable vibrations and one of the effective countermeasures is to connect stay cables by cross-ties to form a cable network. However, the dynamic behaviour of a cable network is not clearly understood and most of the current designs are based on physical tests and numerical simulations. In this thesis, an effort has been made to understand the in-plane free vibration of a cable network. Three analytical models have been developed to investigate the in-plane free vibration of a basic cable network with a rigid transverse cross-tie, a general cable network with a single line of rigid transverse cross-ties, and a basic cable network with a flexible transverse cross-tie. The key system parameters of a cable network have been identified, which include the segment ratio, the frequency ratio, the mass-tension ratio, the length ratio, the cross-tie flexibility parameter, and the total number of interconnected cables. Extensive parametric study has been conducted to evaluate the role of each parameter in affecting the performance of a cable network. All the analytical model results are verified by numerical simulations using the Finite Element software ABAQUS 6.9.

DEDICATION

I would like to dedicate my work to my mother and the memory of my father who made this work easy for me, and to my wife, Riffat, my children, Usama, Sara and Talha.

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TABLE OF CONTENTS

DECLARATION OF ORIGINALITY	iii
ABSTRACT	v
DEDICATION	vi
ACKNOWLEDGEMENTS	vii
LIST OF TABLES	xi
LIST OF FIGURES	xii
NOMENCLATURE	xv
CHAPTER 1 Introduction	1
1.1 Background	1
1.2 Motivations	5
1.3 Objectives	6
1.4 Organization	7
REFERENCES	8
CHAPTER 2	10
Analytical Study on In-plane Modal Behavior of Stay Cables Connected by Cross-ties:Part I: Basic Cable Network	10
2.1 Introduction	10
2.2 In-plane Free Vibration of a Basic Cable Network	14
2.2.1 Description of the system	14
2.2.2 In-plane free vibration of a single taut cable	16
2.2.3 Formulation of system equation	17
2.3 Finite Element Model	21
2.4 Applications to basic cable networks with different configurations	22
2.4.1 Twin-cable network	22
2.4.2 Symmetric two-cable network with unequal length main cables	34
2.4.3 Asymmetric two-cable network with a rigid cross-tie at arbitrary location	43
2.5 CONCLUSIONS	48
REFERENCES	51

CHAPTER 3	54
Analytical Study on In-plane Modal Behavior of Stay Cables Connected by Cross-ties: Part II: General Cable Network	54
3.1 Introduction	54
3.2 In-plane Free Vibration of a General Cable Network	56
3.2.1 Description of the system	56
3.2.2 Formulation of the system equation	57
3.3 Applications to General Cable Networks with Different Configurations	61
3.3.1 Symmetric SMT two-cable network with a single line of rigid cross-ties at mid-span	62
3.3.2 Symmetric SMT four-cable network with a single line of rigid cross-ties at mid-span	66
3.3.3 Symmetric DMT four-cable network with a single line of rigid cross-ties at mid-span	70
3.4 Parametric Study	76
3.4.1 Length ratio	78
3.4.2 Segment ratio	82
3.4.3 Frequency ratio	86
3.4.4 Mass-tension ratio	89
3.4.5 Number of main cables	92
3.5 CONCLUSIONS	93
REFERENCES	96
CHAPTER 4	99
In-plane Free Vibration of a Two-cable Network with Flexible Cross-tie	99
4.1 Introduction	99
4.2 Formulation of analytical model	101
4.2.1 Description of the system	101
4.2.2 Derivation of the system equation	102
4.3 Applications to cable networks with different configurations	106
4.3.1 Twin-cable network with a flexible cross-tie at arbitrary location	106
4.3.2 Symmetric SMT two-cable network with flexible cross-tie at mid-span	114
4.3.3 Symmetric DMT two-cable network with a flexible cross-tie at mid-span	118
4.4 Conclusion	121
CHAPTER 5 Conclusions	124

5.1	Summary	124
5.2	Basic cable network with a transverse rigid cross-tie	124
5.3	General cable network with a single line of rigid transverse cross-ties	125
5.4	Basic cable network with a transverse flexible cross-tie	127
5.5	Suggestions for future work	128
VITA AUCTORIS		124

LIST OF TABLES

Table 2-1	In-plane modal properties of a twin-cable network with a rigid cross-tie at mid-span	32
Table 2-2	In-plane modal properties of twin-cable network with a rigid cross-tie at quarter span	33
Table 2-3	In-plane modal properties of a symmetric SMT two-cable network With unequal cable lengths and a rigid cross-tie at mid-span	38
Table 2-4	In-plane modal properties of a symmetric DMT two-cable network with unequal cable lengths and a rigid cross-tie at mid-span	41
Table 3-1	In-plane modal properties of a symmetric SMT two-cable network with system parameters as frequency ratio $\eta_2=0.833$ and segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4)	65
Table 3-2	In-plane modal properties of a symmetric SMT and a symmetric DMT four-cable network ($\eta_i=0.833$, $i=2, 3, 4$) with a single line of rigid cross-ties at mid-span ($\varepsilon_j=1/2$, $j=1$ to 8)	74
Table 4-1	Comparison of the in-plane modal properties of a symmetric twin-cable network with rigid ($\psi_1=0$) and flexible cross-tie ($\psi_1=1.0$) at quarter span	112
Table 4-2	In-plane modal properties of a symmetric SMT two-cable network with system parameters as frequency ratio $\eta_2=0.88$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi_1=1.0$	117
Table 4-3	Comparing in-plane modal properties of a symmetric SMT cable network with rigid and flexible cross-tie with system parameters as frequency ratio $\eta_2=0.88$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi_1=1.0$	120

LIST OF FIGURES

Figure 2-1	Schematic diagram of the mathematical model of a basic cable network	15
Figure 2-2	Schematic diagram of a symmetric twin-cable network with rigid cross-tie at arbitrary point	22
Figure 2-3	Non-dimensional modified system frequency, Ω / π , as a function of the non-dimensional cross-tie position, ϵ , for a twin-cable SMT network	24
Figure 2-4	First ten modes of a symmetric twin-cable system with a rigid cross-tie at mid-span	28
Figure 2-5	First ten modes of a symmetric twin-cable system with a rigid cross-tie at quarter span from the left end	30
Figure 2-6	Symmetric SMT two-cable network with unequal length main cables and a rigid cross-tie at mid-span	34
Figure 2-7	First ten modes of a symmetric SMT two-cable network with unequal length main cables and a rigid cross-tie at mid-span	36
Figure 2-8	Non-dimensional modified frequency, Ω/π , as a function of frequency ratio parameter, η , for a symmetric SMT two-cable network	37
Figure 2-9	First ten modes of a symmetric DMT two-cable network	41
Figure 2-10	Asymmetric two-cable network with a rigid cross-tie at arbitrary location	43
Figure 2-11	2-11 First ten modes of an asymmetric two-cable network ($H1=H2$, $m1=m2$, $L1=0.8L2$, rigid cross-tie locates at one-fifth from the left end)	46
Figure 3-1	Schematic diagram of the mathematical model of an extended cable network	56
Figure 3-2	Schematic diagram of the mathematical model of a general cable network	61
Figure 3-3	Schematic diagrams of three SMT cable networks having the same system parameters	63
Figure 3-4	First ten modes of a symmetric SMT two-cable network (network-2)	65
Figure 3-5	First ten modes of a symmetric SMT four-cable network	69
Figure 3-6	First ten modes of a symmetric DMT four-cable network	73
Figure 3-7	Schematic diagram of the mathematical model of a two-cable network	76
Figure 3-8a	Non-dimensional modified frequency, Ω/π , as a function of the length ratio	78

	parameter, λ , for three different cross-tie positions (Symmetric DMT cable network, $\gamma_2=0.67$)	
Figure 3-8b	Non-dimensional modified frequency, Ω/π , as a function of the length ratio parameter, λ , for three different cross-tie positions (Symmetric SMT cable network, $\gamma_2=1.0$)	79
Figure 3-8c	Non-dimensional modified frequency, Ω/π , as a function of the length ratio parameter, λ , for three different cross-tie positions (Symmetric DMT cable network, $\gamma_2=1.5$)	79
Figure 3-9a	Non-dimensional modified frequency, Ω/π , as a function of the segment ratio parameter ε for three symmetric SMT cable networks ($\gamma_2=1.0$)	83
Figure 3-9b	Non-dimensional modified frequency, Ω/π , as a function of the segment ratio parameter ε for three symmetric DMT cable networks ($\gamma_2=0.667$)	83
Figure 3-9c	Non-dimensional modified frequency, Ω/π , as a function of the segment ratio parameter ε for three symmetric DMT cable networks ($\gamma_2=1.5$)	84
Figure 3-10a	Non-dimensional modified frequency, Ω/π , as a function of the frequency ratio parameter, η , for four symmetric SMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at mid-span ($\varepsilon=1/2$)	87
Figure 3-10b	Non-dimensional modified frequency, Ω/π , as a function of the frequency ratio parameter, η , for four symmetric SMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at 1/3 span ($\varepsilon=1/3$)	87
Figure 3-11a	Non-dimensional modified frequency, Ω/π , as a function mass-tension parameter, γ , for four symmetric DMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at mid-span ($\varepsilon=1/2$)	90
Figure 3-11b	Non-dimensional modified frequency, Ω/π , as a function mass-tension parameter, γ , for four symmetric DMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at 1/3 span ($\varepsilon=1/3$)	90

Figure 4-1	Schematic diagram of the mathematical model of a basic cable network with flexible cross-tie	100
Figure 4-2	Schematic diagram of a symmetric twin-cable network with flexible cross-tie at arbitrary point	106
Figure 4-33	Evolution of first three right segment (RS) modes of a symmetric twin-cable network with cross-tie located at quarter span and flexibility parameter ψ_1 varies from 0 to 1.0	108
Figure 4-4	Transformation of first ten modes of a symmetric twin-cable network as flexibility parameter Ψ_1 varies from 0 to 1.0 and cross-tie locates at quarter span	111
Figure 4-5	Symmetric SMT two-cable network with unequal length main cables and flexible cross-tie	113
Figure 4-6	First ten modes of a symmetric SMT cable network with system parameters as frequency ratio $\eta_2=0.883$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi_1=1.0$	116
Figure 4-7	First ten modes of a symmetric DMT cable network with system parameters as Frequency ratio $\eta_2=0.88$, mass-tension ratio $\gamma_2=1.25$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter as $\psi_1=1.0$	119

NOMENCLATURE

A	cross-sectional area of main cable (s)
A_1, A_2, A_3, A_4	coefficients of cable segment displacement
$B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8$	coefficients of cable segment displacement
C, D	coefficients of cross-tie displacement
E	modulus of elasticity
EA	axial rigidity
f	modified fundamental frequency of the coupled cable network
f_i	fundamental frequency of the i^{th} main cable in the cable network
H_1, H_2, H_3, H_4	pre-stressed force in the main cables in the cable network
H_i	pre-stressed force in the i^{th} main cable in the cable network
h	additional cable tension due to dynamic vibration
L_1, L_2, L_3, L_4	span lengths of the main cables in the cable work
L_i	span length of the i^{th} main cable in the cable work
L_c	length of the cross-tie in the cable work
$l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8$	segment lengths of the main cables in the cable network
m_1, m_2, m_3, m_4	mass density of the main cables in the cable network
m_i	mass density of the i^{th} main cable in the cable network
O_2	horizontal offset of the second main cable
T	cable tension under static condition.
u_1, u_2	longitudinal displacement function of the cross-tie $u(x, t)$
$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$	in-plane transverse displacement function of the cable segments
x	horizontal distance measured from the support as indicated
$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$	horizontal distance measured along the cable segment as indicated

α_i	wave number of the i^{th} main cable in the cable network. $\alpha = \sqrt{\frac{m_i}{H_i}} \omega$
β	wave number of the cross-tie. $\beta = \sqrt{\frac{m}{EA}} \omega$
λ^2	inextensibility parameter
θ	angle of inclination of the cable to the horizontal axis
ω	circular modified fundamental frequency of the coupled cable network
ω_i	fundamental frequency of the i^{th} main cable in the cable network
τ	additional cable tension induced by cable vibration
ε	frequency ratio parameter
η_i	frequency ratio parameter related to the ratio of frequency of the longest cable in the network to the frequency of the i^{th} cable in the network $\eta_i = f_1/f_i$
η_c	frequency ratio parameter of the cross-tie
ε	segment ratio parameter
γ	mass-tension ratio parameter related to the ratio of ‘mass-tension’ product of the given cable to the longest cable in the network
γ_i	mass-tension ratio parameter of the i^{th} cable in the cable network $\gamma_i = \sqrt{H_i m_i / H_1 m_1}$
Ω	Non-dimensional frequency of the modified coupled cable network. $\Omega = \pi \frac{\omega}{\omega_1}$

CHAPTER 1

Introduction

1.1 Background

The idea of using inclined cables to support a bridge span is not new but the methodology is. The Egyptians used the same idea in their sailing ships (Troitsky, 1977). In the early 17th and 18th century, different simple bridges having a deck supported by inclined bars or cables (Troitsky, 1977) were designed. In spite of all these innovative ideas, cable-stayed bridges were not successful until the late 20th century.

One of the main achievements of the recent advancement in bridge engineering is the design of cable-stayed bridges with much longer span length due to the ambitions of human to cross wider space. The complex behaviour of cable-stayed bridges when subjected to dynamic loads, such as wind, traffic or seismic excitations is a long standing issue in bridge industry. The nonlinear interaction between local cable vibrations and bridge deck vibrations can lead to very large amplitude cable vibrations or high amplitude of deck vibrations. These can in turn cause problems related to structural safety, user discomfort or fatigue damage. Besides, wind-induced cable vibrations are identified as one of the critical dynamic problems associated with cable-stayed bridge at present. The initiative and the amplitude of excitation depend on the flow characteristics of wind, geometry and dynamic properties of the cable. Based on their mechanisms, wind-induced cable vibrations can be categorized as rain-wind-induced vibration, vortex-induced vibration, buffeting, wake galloping, high-speed vortex excitation and dry inclined cable galloping.

Suppressing vibrations of bridge stay cables is of prime importance since stay cables are key structural elements of a cable-stayed bridge. Frequent and/or excessive vibrations of stay

cables would have a considerable impact on the serviceability and life span of the entire bridge. Therefore, to control cable vibrations, different countermeasures are adopted, which can be classified as the aerodynamic type and the mechanical type. The aerodynamic type of countermeasures aims at changing the aerodynamic behaviour of the stay cables by modifying its cross-sectional shape. Experimental results show that by using spiral wire around the cable surface (Zhan et al, 2008), dimpled surface (Verlogieux, 1998) and the helical wire whirling surface (Bosdogianni & Olivari, 1996), the amplitude of cable vibration could be reduced significantly. On the other hand, the mechanical type of countermeasures is directed to enhance damping property and stiffness of the cable(s). External dampers installed near the cable-deck anchorage are used to help dissipate kinetic energy of an oscillating cable and increase the structural damping of the attached cable (Pacheco et al, 1993; Krenk, 2000; Tabatabai and Mehrabi, 2000; Zhan et al, 2008; Cheng et al, 2010). The cross-ties or transverse secondary cables, used to connect a problematic stay cable with its neighbour(s) and thus form a cable network, serves to increase the stiffness and thus natural frequencies of the cable, so that the critical wind velocity at which aerodynamic instability of the cable would initiate can be increased. The cross-ties can also be used to reduce cable sag variation among stays of different lengths (Gimsing, 1993). So far, cross-ties have been successfully used on a number of cable-stayed bridges to control cable vibrations, such as the Faro Bridge in Denmark, the Normandie Bridge in France, the Yobuko Bridge in Japan (Virlogeux, 1998), the Fred Hartman Bridge (Caracoglia and Jones, 2005b) and the Dames Point Bridge in USA (Kumarasena et al., 2007). After installing the cross-ties, no problematic cable vibrations have been reported. The application of cross-ties not only increases the in-plane stiffness of the connected cables, but also

adds extra damping to the system and eliminates some of the lower modes of vibration that could be excited.

However, the cross-tie solution is proposed more recently, studies on the mechanisms and dynamic behaviour of a cable network system is still limited.

Ehsan and Scanlan (1989) used the finite element approach to understand the behaviour of a cable network. According to their study, the main function of the cross-ties in a cable network is to help transferring energy from a more vulnerable cable to its neighbours. Virlogeux (1998) reported an unexpected behaviour of cross-ties on Foro Bridge in Denmark. He addressed the issues of transverse cable stiffness, cable tension and cable internal damping. He suggested that equivalent static analysis sometimes could be efficient in design recommendation.

Yamaguchi and Nagahawatta (1995) performed a set of physical tests on a simple cable network using two cables, with different lengths, connected through a transverse cross-tie. It was found that the fundamental frequency of the modified cable network was higher than that of the top individual cable. It was also found that the modal damping of their simple cable network was always greater than that of a single main cable without cross-tie. This modal damping increment was found to be more significant with flexible (soft) cross-ties as compare to the rigid (stiff) ones.

Caracoglia and Jones (2005a, 2005b) developed an analytical model to study the in-plane free vibration of a cable network. In their model, the taut cable assumption was applied to the main cables and the cross-ties were simplified as either rigid or linear spring cross-ties. In their problem formulation, the equations of motion of the main cables were considered but the equations of motion of the cross-ties were not. However, the system equation was solved numerically. The key system parameters of a cable network were not clearly identified in the

work, and their roles on the dynamic performance of a cable network were not investigated. In a subsequent piece of work (Caracoglia and Zuo, 2009), the same approach was applied to the cable networks with different configurations. But again, a clear reveal of the important system parameters and their effects on the behaviour of a cable network was lacking.

Bosch and Park (2005) simulated the performance of stay cables with cross-ties using the finite element approach. Among their findings, it is interesting to note that the effectiveness of the cross-ties depends on the deployment geometry, the quantity, the size, and the anchorage conditions. It was also observed that when main cables were connected with cross-ties, local modes which were densely populated over a narrow band of frequency range, were induced. In addition, the simulation results indicated that the combined use of cross-ties and external dampers would not necessarily earn the cumulative benefits of both when they are applied separately.

Sun et al (2007) performed a set of physical tests on a scaled model of cable network using three cables connected through a cross-tie. The different factors such as the cross-tie stiffness, the tensioning method and the pretension of the cross-ties were considered to see the behaviour of their cable network. It was among their findings that stiff type of cross-tie mainly contributed to enhance the in-plane stiffness of a cable network while the soft type of cross-tie was more effective in increasing the system damping.

More recently, a simplified analytical model was developed by Zhou et al (2011) to study the free vibration of a cable network. In this model, only the target cable was included in the model and the cross-ties were modeled as linear springs.

1.2 Motivations

Though the strategy of using cross-ties to suppress the unfavourable cable vibrations has already been implemented on some cable-stayed bridges and a number of successful practical examples were reported in the literature (Faro Bridge in Denmark, the Normandie Bridge in France, the Yobuko Bridge in Japan, the Fred Hartman Bridge and the Dames Point Bridge in USA), the dynamic behaviour of such a cable network system is not fully understood.

Majority of the existing studies were based on either physical tests or numerical simulations. In the knowledge of the author, only few studies (Caracoglia and Jones, 2005a; 2005b; Zhou et al, 2011) attempted analytically. The model developed by Zhou et al (2011) was really simple and had the major flaw by ignoring influence of the neighbouring cables. The model proposed by Caracoglia and Jones (2005a), though easy to understand, the form of the derived characteristic equations corresponding to different network configurations were complicated for understanding the behaviour of a cable network. As far as the finite element simulation is concerned, it is an effective tool to find the final results but needs a lot of effort in the parametric study where the redistribution of nodes and elements of a cable network is required. Further, the roles of different system parameters cannot be clearly explained and the exact relations between them are hard to conclude.

It is also noted that studies of a general cable network including n number of cables connecting through cross-tie are not available in the literatures. Therefore, there is a great need to develop an analytical model of a general cable network that can include all the essential components in this structural system and identify the key system parameters as well as their roles in the behaviour of the cable network.

All these above motivated the present study, of which an analytical model of a general cable network consisting of n horizontal cables and a single line of transverse cross-ties will be developed and the system equation will be solved analytically. The key system parameters will be identified. The impact of different system parameters on the in-plane modal behaviour of a cable network will be extensively investigated.

1.3 Objectives

The objectives of the current study are proposed as follows:

1. Propose an analytical model to describe the in-plane modal behaviour of a basic cable network, which consists of two stay cables connected through a single rigid transverse cross-tie.
2. Develop a finite-element model for the basic cable network to verify the proposed analytical model.
3. Apply the proposed analytical model to study in-plane modal behaviour of basic cable networks with different configurations.
4. Extend the analytical model of a basic cable network to a general cable network consisting of n horizontally laid main cables connected by a single line of rigid transverse cross-ties. Develop system equation and analytical solutions. Validate the general cable network model using finite element approach.
5. Identify the key system parameters in a general cable network. Conduct parametric study to extensively explore the role of each system parameters in affecting the dynamic behaviour of a cable network.
6. Develop an analytical model of a basic cable network of which two horizontally laid main cables are connected by a flexible cross-tie. Compare the results with the rigid

cross-tie model and study the effect of cross-tie flexibility on the dynamic behaviour of a cable network.

1.4 Organization

The material presented in this thesis is divided into five chapters. The first chapter, Chapter 1, includes an introduction of the studied problem, a brief review of the existing studies, a discussion about the motivations that led to the current study, and the objectives to be achieved. Chapters 2, 3, and 4 are prepared in the form of journal papers, with each contains its own introduction, literature review, methodology, main results, and conclusions. Chapter two (Journal paper 1) is focused on developing an analytical model of a basic cable network with a rigid transverse cross-tie, and its application to basic cable networks of different configurations. Chapter 3 (Journal paper 2) is dedicated to a general cable network with a single line of rigid transverse cross-ties. An analytical model describing the in-plane free vibration of a general cable network is proposed, from which the key system parameters are identified. An extensive parametric study is conducted to investigate the role of each parameter on affecting the modal behaviour of a network system. In Chapter 4 (Journal paper 3), the flexibility of cross-tie is the main interest. An analytical model of a basic cable network with flexible cross-tie is developed. The role of cross-tie flexibility on the in-plane stiffness of a cable network is discussed. The last chapter, Chapter 5, concludes the present work and also provides recommendations for the future work in this area.

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CHAPTER 2

Analytical Study on In-plane Modal Behavior of Stay Cables Connected by Cross-ties: Part I: Basic Cable Network

2.1 Introduction

One main achievement in the recent advancement in bridge engineering is the design of cable-stayed bridges with much longer span length due to the ambitions of human to cross wider space and the rapid development of materials and technology. Light weight of structural components and the use of longer spans result in more slender elements and structures which are susceptible for vibrations under various types of dynamic excitations. A typical example is the cables on cable-stayed bridges. They run directly from a bridge deck up to a tower, forming a unique "A" shape. Mainly due to their high flexibility, light mass and very low inherent damping, these cables are unable to dissipate much of the excitation energy, which renders large amount of accumulation.

The complex behavior of cable-stayed bridges when subjected to dynamic loads, such as wind, traffic or seismic excitation, is a long standing issue in bridge engineering. The nonlinear interaction between local cable vibrations and bridge deck vibrations can lead to very large amplitude cable vibrations or high amplitude of deck vibrations (e.g. Lilien and Pinto da Costa, 1974; Macdonald and Georgakis, 2002; Caetano and Cunha, 2003; Sun et al, 2003). These can in turn cause problems related to structural safety and user discomfort.

Different solutions have been used on site to suppress undesired cable vibrations. Surface treatment is applied to cables to change their aerodynamic features. A study (Zhan et al, 2008) on rain-wind induced cable vibration showed that by using spiral wire around the cable surface, the upper rivulet could not form on the surface of the cable model and therefore reduced the cable vibration amplitude to one-tenth. Similarly, on Higashi-Kobe Bridge, stay cable vibration was

suppressed by modifying cable-surface with parallel longitudinal projections in order to prevent the formation of water rivulets (Matsumoto et al, 1992). Besides, mechanical types of solutions including installation of external dampers and connection of a few cables by cross-ties have been implemented on site with different levels of effectiveness.

The structural behaviour of a cable when attached transversely with an external damper and the optimum design of such a cable-damper system have been extensively studied by many researchers (e.g. Pacheco et al, 1993; Krenk, 2000; Tabatabai and Mehrabi, 2000; Main and Jones, 2002; Fujino and Hoang, 2008; Cheng et al, 2010). However, the mechanism of cross-tie solution is yet well understood. When cross-tie(s) are applied on site, the cables which are vulnerable (or have experienced) large amplitude vibrations are connected to their neighboring cables through transverse secondary cables or cross-ties to form a cable network. Such a strategy aims at reducing the effective length of the problematic cable to increase its in-plane stiffness and natural frequencies. Besides this, the adoption of cross-ties also adds extra damping to the target cable which would help to eliminate excitation of vibrations in some lower modes. The cable sag variation among cables of different lengths is also found to be reduced with the introduction of cross-ties (Gimsing, 1993).

So far, cross-ties have been successfully used on a number of cable-stayed bridges to control cable vibrations, such as the Faro Bridge in Denmark, the Normandie Bridge in France, the Yobuko Bridge in Japan (Virlogeux, 1998), the Fred Hartman Bridge (Caracoglia and Jones, 2005b) and the Dames Point Bridge in USA (Kumarasena et al, 2007). After installing the cross-ties, no problematic cable vibrations have been reported. However, studies on the dynamic behaviour of this type of structural system are still limited.

Ehsan and Scanlan (1989) used the finite element approach for the solution of a three-dimensional cable network problem and studied the redistribution of oscillation energy in a cable network consisting of a number of stay cables connected by cross-ties. According to their study, the main function of the cross-ties in a cable network is to help transferring energy from a problematic cable to its neighbours.

Virlogeux (1998) described the unexpected behaviour of cross-ties on the Normandie Bridge in France where one of the cross-ties was broken. It was not clear what caused the failure of this cross-tie. However, when these cross-ties were replaced by stronger ones with higher tension, the performance was found to be satisfactory. In another case reported by the same author, the installation of extremely low damping cross-ties was found to successfully suppress the rain-wind-induced vibrations of bridge stay cables.

Yamaguchi and Nagahawatta (1995) performed a set of physical tests on a simple cable network, of which two main cables of different lengths were connected through two transverse cross-ties symmetrically. It was found that the fundamental frequency of the cable network was higher than that of the top individual main cable (the longer ones) and increased monotonically with the prestress in the cross-tie. The modal damping of the cable network was always greater than that of a single main cable without cross-tie. This modal damping increment was found to be more significant when more flexible (soft) cross-ties were used.

Caracoglia and Jones (2005a) developed an analytical procedure to study the in-plane free vibration of a cable network. In the formulation, the taut cable assumption was applied to the main cables and the cross-tie was simplified as either rigid or a linear spring. The approach was implemented numerically for a number of simplified two-cable networks and a more complicated cable network on a real cable-stayed bridge (Caracoglia and Jones, 2005b). However, the

important system parameters in a typical cable network cannot be identified clearly through numerical simulation. In addition, the high cost associated with reconstruction of numerical model limited the capability of the approach for an extensive parametric study. In a more recent work (Caracoglia and Zuo, 2009), this analytical procedure was applied to cable networks of different configurations to determine the effectiveness of the cross-tie solution.

Bosch and Park (2005) simulated the performance of a group of stay cables connected by cross-ties using the finite element software, SAP 2000. Interestingly, it was found that the effectiveness of the cross-ties depended on their deployment geometry, the quantity, the size, and the anchorage conditions with the main cables. It was observed from the numerical simulation that when cables were connected by transverse cross-ties, vibration modes densely populated over a narrow band of frequency range would be induced. In addition, it was found that the combined use of cross-ties and external dampers would not necessarily produce the sum of benefits when they are used separately.

In an experimental study by Sun et al (2007), a scaled cable network model of three main cables connected by a transverse cross-tie was tested. The effect of different factors such as the cross-tie stiffness, the tensioning method and the pretension of the cross-ties were evaluated. It was found that while the stiff type of cross-tie mainly contributed to enhance the in-plane stiffness of a cable network and thus its modal frequencies, the soft type of cross-tie is more effective in increasing the system damping.

A simplified analytical model was proposed lately by Zhou et al (2011) to study the free vibration of a cable network. Only the target cable was included in the model, and the effects of cross-ties were modeled as linear springs.

In majority of the previous studies, either physical tests or numerical simulations were conducted to explore the dynamic behaviour of a cable network. In the knowledge of the authors, very few studies (e.g. Caracoglia and Jones, 2005a; 2005b; Zhou et al, 2011) attempted analytically. In the case of Caracoglia and Jones (2005a; 2005b), the proposed analytical procedure was implemented numerically. Thus, the key system parameters as well as their contributions to the performance of a cable network could not be clearly explained. Nevertheless, these information will be essential to the design of such a vibration control strategy.

In the present paper, an analytical model of a basic cable network consisting of two horizontally laid taut cables connected by a rigid vertical cross-tie will be developed. The solution to the system equation will be derived analytically, based on which the key system parameters and their relations with the dynamic response of the cable network can be explicitly identified from the system characteristic equation. Examples of a number of two-cable networks with typical configurations and various system properties will be presented. The in-plane modal behaviour of these basic cable networks will be extensively examined. The proposed analytical model and approach will be extended to a more general cable network in a companion paper (Ahmad and Cheng, 2012).

2.2 In-plane Free Vibration of a Basic Cable Network

2.2.1 Description of the system

The proposed mathematical model of a basic cable network comprised of two horizontally laid main cables having different lengths L_1 and L_2 , with L_1 being the length of the longer one. Both cables are connected through a vertical cross-tie of length L_c , which divides each cable into two segments, as shown in Figure 2-1. The mass per unit length of the two cables is m_1 and m_2 , respectively, and the pre-stressing force is H_1 and H_2 , respectively. The horizontal

offset of the second cable on the left end is denoted by O_L and that on the right by O_R . The cross-tie is located at l_1 from the left end of cable AB. The downward vertical displacements of the main cables and the cross-tie are defined as positive.

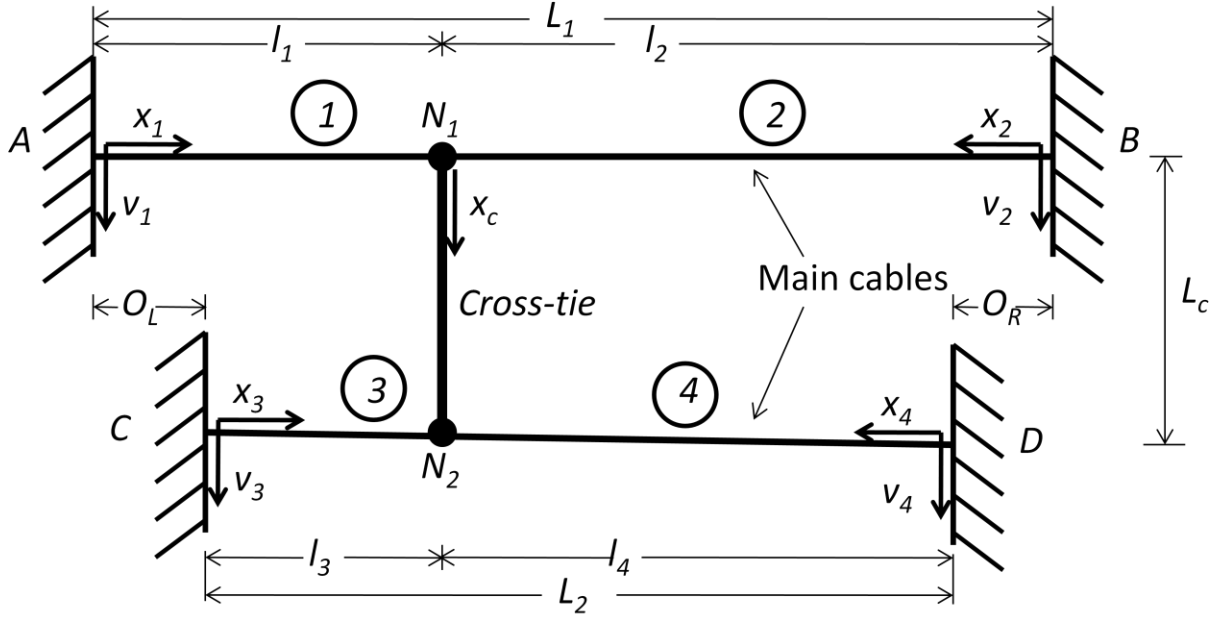


Figure 2-1 Schematic diagram of the mathematical model of a basic cable network

The analytical model of a basic cable network is developed based on the following assumptions: a) The main cables are idealized as taut cables; b) The additional dynamic tensions in the main cables are neglected; c) The main cables are fixed at both ends; d) The main cables have the in-plane transverse vibration as their dominant motion, the longitudinal vibration is neglected; e) The cross-tie has the longitudinal vibration as its dominant motion, the transverse motion is neglected; f) The cross-tie is rigid. The last assumption is made not only to simplify the formulation and solution, but considers the fact that the fundamental frequency of a cable network will only be slightly decreased when choosing a flexible cross-tie as oppose to a rigid one used on real bridges (Caracoglia and Jones, 2005b).

2.2.2 In-plane free vibration of a single taut cable

As can be seen from Figure 2-1, when the cable network vibrates in the vertical direction, the two main cables are expected to oscillate transversely, where the cross-tie will be subjected to axial vibration. Therefore, the equation of motion describing the in-plane transverse vibration and axial vibration of a single taut cable will be briefly reviewed first.

Since the additional cable tension due to its motion is neglected, the in-plane free vibration of a single taut cable in the transverse direction can be expressed as (Irvine, 1974)

$$H \frac{\partial^2 v}{\partial x^2} = m \frac{\partial^2 v}{\partial t^2} \quad (2-1)$$

where H , m and v are the pre-stressed force, the unit mass and the in-plane transverse displacement in main cable(s), respectively.

whereas that for the in-plane longitudinal motion is (Humar, 2001)

$$EA \frac{\partial^2 u}{\partial x^2} = m \frac{\partial^2 u}{\partial t^2} \quad (2-2)$$

where E , A and u are the modulus of elasticity, the cross-sectional area the longitudinal displacement in secondary cable(s) or cross-tie(s), respectively.

By applying the Bernoulli-Fourier method to the in-plane transverse displacement $v(x, t)$ to separate the time-dependent and spatial coordinate variables, gives $v(x, t) = \bar{v}(x)\sin(\omega t)$. Its second derivatives with respect to time t and spatial coordinate x are respectively

$$\ddot{v}(x, t) = -\omega^2 \bar{v}(x)\sin(\omega t)$$

and

$$v''(x, t) = \bar{v}''(x)\sin(\omega t)$$

Substitute the above two equations into Eq. (2-1) and simplify it, gives

$$\bar{v}''(x) + \alpha^2 \bar{v}(x) = 0 \quad (2-3)$$

where $\alpha^2 = m\omega^2/H$ and ω is the circular frequency of cable vibration. Solution to Eq. (2-3) is in the form of

$$\bar{v}(x) = A\cos(\alpha x) + B\sin(\alpha x) \quad (2-4)$$

where A and B are constants, which can be determined from boundary conditions.

Since the cable is suspended horizontally and fixed at both ends, $\bar{v}(0) = 0$ and $\bar{v}(L) = 0$. Applying these boundary conditions to Eq. (2-4) gives $A=0$ and $B\sin(\alpha L) = 0$. The constant B cannot be zero in order for a non-trivial solution, thus

$$\bar{v}(x) = B\sin(\alpha x) \quad (2-5)$$

Similarly, the in-plane axial displacement $u(x, t)$ in Eq. (2-2) can be expressed as $u(x, t) = \bar{u}(x)\sin(\omega t)$, by separating the variables. Its second derivatives with respect to time t and spatial coordinate x are respectively

$$\ddot{u}(x, t) = -\omega^2 \bar{u}(x)\sin(\omega t)$$

and

$$u''(x, t) = \bar{u}''(x)\sin(\omega t)$$

Substitute the above two equations into Eq. (2-2), yields

$$\bar{u}''(x) + \beta^2 \bar{u}(x) = 0 \quad (2-6)$$

where $\beta^2 = m\omega^2/(EA)$ and ω is the circular frequency of cable vibration. Solution to Eq. (2-6) is in the form of

$$\bar{u}(x) = C\cos(\beta x) + D\sin(\beta x) \quad (2-7)$$

where C and D are constants determined from boundary conditions.

2.2.3 Formulation of system equation

In the basic cable network model shown in Figure 2-1, there are four main cable segments vibrating in the transverse direction and one cross-tie in the axial direction. Applying Eqs. (2-5) and (2-7) to describe their motion, and also introducing the frequency ratio $\eta_i = f_1/f_i$

and $\eta_c = f_1/f_c$, and the non-dimensional system frequency $\Omega = \pi f/f_1$, where f_i and f_c are the fundamental frequency of the i^{th} ($i=1, 2$) cable and the cross-tie, respectively and f is the fundamental frequency of the cable network yet to be determined. The wave numbers α and β in Eqs. (2-5) and (2-7) can be expressed as $\alpha_i = \Omega\eta_i/L_i$ and $\beta = \Omega\eta_c/L_c$. Thus, Eqs. (2-5) and (2-7) can be rewritten as

$$\bar{v}_{2i-1}(x_{2i-1}) = B_{2i-1} \sin(\Omega\eta_i x_{2i-1}/L_i) \quad i=1, 2 \quad (2-8a)$$

$$\bar{v}_{2i}(x_{2i}) = B_{2i} \sin(\Omega\eta_i x_{2i}/L_i) \quad i=1, 2 \quad (2-8b)$$

$$\bar{u}_c(x_c) = C \cos(\Omega\eta_c x_c/L_c) + D \sin(\Omega\eta_c x_c/L_c) \quad (2-8c)$$

where i is the numbering of the main cables; B_{2i-1} and B_{2i} are the shape function constants of the four main cable segments shown in Figure 2-1, and C, D are the shape function constants of the cross-tie. It is worth noting that based on the definition, frequency ratio η_1 of the first main cable (the longer one) is 1. Eqs. (2-8a) and (2-8b) represent respectively the response of the left and the right segments of the two main cables in Figure 2-1, whereas Eq. (2-8c) is for the cross-tie. The six unknown shape function constants B_1, B_2, B_3, B_4, C and D can be determined using the following conditions:

Compatibility conditions

$$\bar{v}_1(l_1, t) = \bar{v}_2(l_2, t); \quad \bar{v}_3(l_3, t) = \bar{v}_4(l_4, t) \quad (2-9a)$$

$$\bar{v}_1(l_1, t) = \bar{v}_3(l_3, t) \quad (2-9b)$$

$$\bar{v}_1(l_1, t) = \bar{u}_1(0, t) \quad (2-9c)$$

$$\bar{u}_1(0, t) = \bar{u}_1(L_c, t) \quad (2-9d)$$

Equilibrium conditions

By isolating the cross-tie $N_1 N_2$ from the network, forces acting on it should satisfy

$$H_1\left(\frac{\partial \bar{v}_1}{\partial x_1}\bigg|_{x_1=l_1} + \frac{\partial \bar{v}_2}{\partial x_2}\bigg|_{x_2=l_2}\right) + H_2\left(\frac{\partial \bar{v}_3}{\partial x_3}\bigg|_{x_3=l_3} + \frac{\partial \bar{v}_4}{\partial x_4}\bigg|_{x_4=l_4}\right) = 0 \quad (9-e)$$

Applying the above compatibility and equilibrium conditions to Eq. (2-8) and expressing the resulting equations in the matrix form, yields

$$\begin{bmatrix} \sin(\phi_1) & -\sin(\phi_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & -\sin(\phi_4) & 0 & 0 \\ \sin(\phi_1) & 0 & -\sin(\phi_3) & 0 & 0 & 0 \\ \sin(\phi_1) & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 - \cos(\phi_c) & \sin(\phi_c) \\ \gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4) & 0 & 0 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2-10)$$

where $\phi_{2i-1} = \Omega \eta_i \varepsilon_{2i-1}$ ($i=1, 2$) applies to the left segments of both main cables with $\varepsilon_{2i-1} = l_{2i-1}/L_i$ being their segment ratios; and $\phi_{2i} = \Omega \eta_i \varepsilon_{2i}$ ($i=1,2$) applies to the right segments of both main cables with the corresponding segment ratios as $\varepsilon_{2i} = l_{2i}/L_i$; $\phi_c = \Omega \eta_c$ applies to the cross-tie and $\gamma_i = \sqrt{H_i m_i / (H_1 m_1)}$ ($i=1, 2$) is the mass-tension ratio parameter of the i^{th} cable.

To obtain non-trivial solution to the system equation, Eq. (2-10), the determinant of the 6×6 coefficient matrix on the left hand side of the equation should be zero, i.e.

$$\begin{vmatrix} \sin(\phi_1) & -\sin(\phi_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & -\sin(\phi_4) & 0 & 0 \\ \sin(\phi_1) & 0 & -\sin(\phi_3) & 0 & 0 & 0 \\ \sin(\phi_1) & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 - \cos(\phi_c) & \sin(\phi_c) \\ \gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4) & 0 & 0 \end{vmatrix} = 0$$

This leads to

$$\begin{aligned} & \gamma_1 \sin(\phi_1) \cos(\phi_2) \sin(\phi_3) \sin(\phi_4) \sin(\phi_c) + \gamma_1 \cos(\phi_1) \sin(\phi_2) \sin(\phi_3) \sin(\phi_4) \sin(\phi_c) \\ & + \gamma_2 \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \cos(\phi_4) \sin(\phi_c) + \gamma_2 \sin(\phi_1) \sin(\phi_2) \cos(\phi_3) \sin(\phi_4) \sin(\phi_c) = 0 \end{aligned}$$

The term $\sin(\phi_c)$ is common in all terms and therefore the above equation can be written as,

$$\begin{aligned} & \sin(\phi_c) [\gamma_1 \sin(\phi_1) \cos(\phi_2) \sin(\phi_3) \sin(\phi_4) + \gamma_1 \cos(\phi_1) \sin(\phi_2) \sin(\phi_3) \sin(\phi_4) \\ & + \gamma_2 \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \cos(\phi_4) + \gamma_2 \sin(\phi_1) \sin(\phi_2) \cos(\phi_3) \sin(\phi_4)] = 0 \end{aligned}$$

Its further simplification generates

$$\sin(\emptyset_c)\{\gamma_1\sin(\emptyset_3)\sin(\emptyset_4)[\sin(\emptyset_1)\cos(\emptyset_2)+\cos(\emptyset_1)\sin(\emptyset_2)] \\ + \gamma_2\sin(\emptyset_1)\sin(\emptyset_2)[\sin(\emptyset_3)\cos(\emptyset_4)+\cos(\emptyset_3)\sin(\emptyset_4)]\} = 0$$

Now, by using $\sin(A+B)$, the above equation can be rewritten as,

$$\sin(\emptyset_c)[\gamma_1\sin(\emptyset_1 + \emptyset_2)\sin(\emptyset_3)\sin(\emptyset_4) + \gamma_2\sin(\emptyset_3 + \emptyset_4)\sin(\emptyset_1)\sin(\emptyset_2)] = 0$$

By definition of \emptyset_i , $\emptyset_1 + \emptyset_2 = \Omega\eta_1\varepsilon_1 + \Omega\eta_1\varepsilon_2$ and $\emptyset_3 + \emptyset_4 = \Omega\eta_2\varepsilon_3 + \Omega\eta_2\varepsilon_4$

By definition of ε_j , $\varepsilon_1 + \varepsilon_2 = \frac{l_1}{L_1} + \frac{L_1-l_1}{L_1} = 1$

Similarly, $\varepsilon_3 + \varepsilon_4 = \frac{l_3}{L_2} + \frac{L_2-l_3}{L_2} = 1$

Substituting the values of $\emptyset_1 + \emptyset_2$ and $\emptyset_3 + \emptyset_4$ yields,

$$\sin(\Omega\eta_c)[\gamma_1\sin(\Omega\eta_1)\sin(\emptyset_3)\sin(\emptyset_4) + \gamma_2\sin(\Omega\eta_2)\sin(\emptyset_1)\sin(\emptyset_2)] = 0 \quad (2-11)$$

This equation is the system equation of a basic cable network where two horizontally suspended taut cables are connected transversely through a rigid cross-tie. It has two sets of solutions. The first set, which can be determined from $\sin(\Omega\eta_c) = 0$, represents the behaviour of the cross-tie. It is of no interest to the present study and therefore is dropped. The second set of solution can be obtained from

$$\gamma_1\sin(\Omega\eta_1)\sin(\emptyset_3)\sin(\emptyset_4) + \gamma_2\sin(\Omega\eta_2)\sin(\emptyset_1)\sin(\emptyset_2) = 0 \quad (2-12)$$

which is the characteristic equation of the basic cable network in Figure 2-1. It can be observed from Eq. (2-12) that the left hand side of the equation is the summation of two terms. Each term is the product of four sub-terms which are the mass-tension ratio parameter of one of the main cables, the Sine term of that main cable and the Sine terms of both segments of the other main cable, respectively.

This equation can be applied to a basic cable network having any arbitrary configurations and properties to study its in-plane modal behaviour and to evaluate how the dynamic response of a cable would be altered once connected to its neighbours through a rigid cross-tie.

2.3 Finite Element Model

To verify the proposed analytical model and the solution for the in-plane modal behaviour of a basic cable network, a finite element model is developed independently. The commercial finite element software package ABAQUS 6.9 is used for this purpose. This software is capable to numerically simulate the performance of structural members in static, free vibration, linear and non-linear dynamic analysis.

In the developed model, both main cables and the cross-tie are modeled as two-dimensional elements with two degrees-of-freedom along transverse and axial directions at each node. The beam element B21 from the ABAQUS element library is chosen to model the main cable. This element is a shear-deformable (Timoshenko) beam and can be used for thin or thick members. The rigid beam element RB2D2 is selected to simulate the behaviour of the rigid cross-tie. It is a two-dimensional, two-node rigid element. All the cables used in the numerical examples in the current study have lengths varying between 48 m to 100 m. From the sensitivity analysis, it was concluded that 50 to 100 elements per cable, depending on the cable length, is adequate to model the cable behaviour.

In order to ensure the accuracy of the results, the material properties of the numerical model must be defined to be compatible with the assumptions made for the analytical model. In the analytical model, the main cables are idealized as taut strings. Therefore, the material properties of the main cables in the finite element model are selected in such a way which would yield the Irvin's inextensibility parameter close to 0 (Irvine, 1974). This parameter was

introduced by Irvin (1974) to describe the static and dynamic behavior of cables. It accounts for the geometric and elastic effects and is defined as $\lambda^2 = (mgL/H)^2 L / [HL_e/(EA)]$, where L_e is the effective length of the cable due to sagging effect. The value of λ^2 varies from a very small number (close to 0), for the taut flat cable, to a very large number (for example, 1000) in the case of an inextensible cable. The cross-tie is idealized as a rigid connector in the analytical model. Therefore, it is taken as the rigid body in the finite element model. The two main cables are fixed at both ends and the initial stress is introduced to simulate the effect of pretension. All boundary conditions are defined when building the finite element model and the initial stress is applied as the initial condition.

For free vibration analysis, the FREQUENCY procedure in the ABAQUS software is adopted, which uses an eigenvalue technique to extract natural frequencies and the corresponding mode shapes of a structure. LANCZOS is used as the Eigen solver. In the output, the eigenvectors are normalized such that the largest displacement value in each vector is unity.

2.4 Applications to basic cable networks with different configurations

In this section, the proposed analytical model of a basic cable network will be applied to two-cable network systems with different configurations and cable properties. Results will be compared with those obtained from numerical simulations using the finite element model described in Section 2.3.

2.4.1 Twin-cable network

2.4.1.1 *Cross-tie at arbitrary location*

In this type of configuration, assume the cross-tie is located at a distance l_1 , with $l_1 \neq l_2$, from the left end of main cable 1, as shown in Figure 2-2.

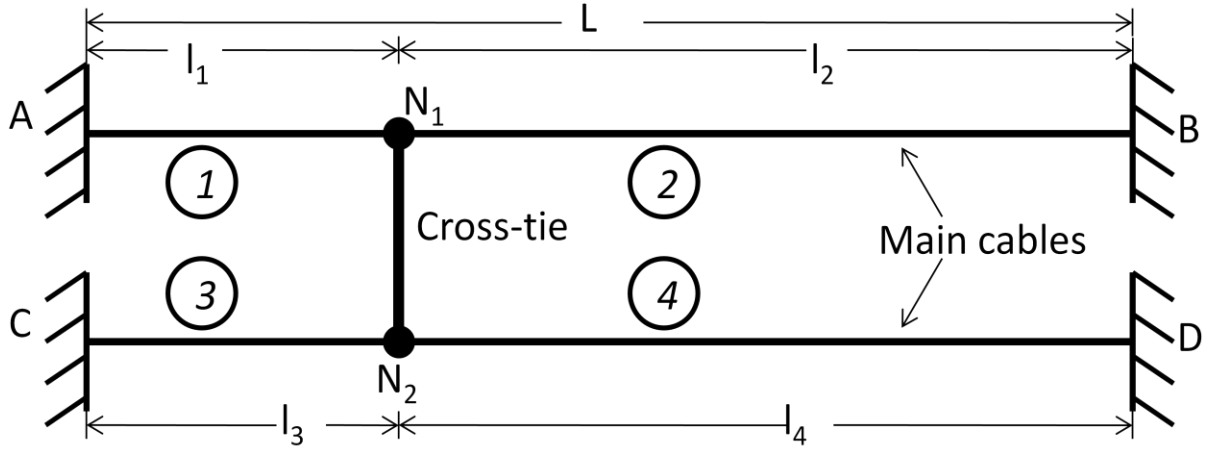


Figure 2-2 Schematic diagram of a symmetric twin-cable network with rigid cross-tie at arbitrary point

Since the main cables are twin cables, so $L_1 = L_2 = L$, $H_1 = H_2$ and $m_1 = m_2$. Therefore, the frequency ratios are $\eta_1 = \eta_2 = 1$, the segment ratios are $\varepsilon_1 = \varepsilon_3 = l_1/L = \varepsilon$ and $\varepsilon_2 = \varepsilon_4 = l_2/L = 1 - \varepsilon$, and the mass-tension ratios are $\gamma_1 = \gamma_2 = 1$. Because the mass-tension ratio parameter γ , is the same for both cables, it is termed as the same-mass-tension ratio or SMT cable network. Substitute these values into Eq. (2-12), gives

$$\sin(\Omega)\sin(\Omega\frac{l_1}{L})\sin(\Omega\frac{l_2}{L}) = 0$$

or

$$\sin(\Omega)\sin(\Omega\varepsilon)\sin[\Omega(1 - \varepsilon)] = 0 \quad (2-13)$$

Obviously, three sets of solution are present for Eq. (2-13). The first set, yielded from $\sin(\Omega) = 0$, generates roots of $\Omega = n\pi$, where $n=1, 2, 3, \dots$. It describes the global modes of the entire cable network system, of which the two main cables vibrate independently at the same natural frequency of a single cable. The odd values of n generate the modified system frequency as $\Omega = \pi, 3\pi, 5\pi, \dots$, and the even values of n generate the modified system frequency as $\Omega = 2\pi, 4\pi, 6\pi, \dots$. Clearly, the expression of the system frequency associated with the global modes, i.e. $\Omega = n\pi$, is not a function of the segment ratio ε . This suggests that if the cross-tie is rigid, the symmetric and the asymmetric global modes of a twin SMT cable network are independent of

the position of the cross-tie. In these global modes, both main cables vibrate in the same shape as that of a single cable at this frequency.

The second and the third sets of solution are functions of the segment ratio ε , i.e. the cross-tie position. They have the form of

$$\Omega = n\pi/\varepsilon \quad (2-14a)$$

and
$$\Omega = n\pi/(1 - \varepsilon) \quad (2-14b)$$

where $n = 1, 2, 3, \dots$. The modified frequency given by Eq. (2-14a) results from the vibrations of cable segment 1 in main cable 1 and its counterpart in main cable 2, i.e. segment 3, while the other two main cable segments are at rest. Similarly, vibrations of cable segments 2 and 4 in Figure 2-2 are associated with the frequency given by Eq. (2-14b). In this study, these two types of local modes are categorized as the left-segment mode (when the left segments are vibrating and the right ones are at rest, designated as ‘LS’ mode) and the right-segment mode (when the right segments are vibrating and the left ones are at rest, designated as ‘RS’ mode). For cross-tie position other than the mid-span, i.e. $\varepsilon \neq 1/2$, it can be seen that the frequencies of lower order modes of one equation in Eq. (2-14) may be the same as those of the higher order ones of the other equation, i.e. they form complimentary pairs. Caracoglia and Jones (2005a) denoted these ‘LS’ and ‘RS’ modes as the ‘complimentary-pseudo-symmetric’ and the ‘pseudo-symmetric’ modes. For cross-tie position at the mid-span, frequencies and mode shapes of the LS modes are the same as those of the RS modes.

The impact of the cross-tie position, i.e. the segment ratio ε , on the system frequency of a twin-cable network with a transverse rigid cross-tie is illustrated in Figure 2-3.

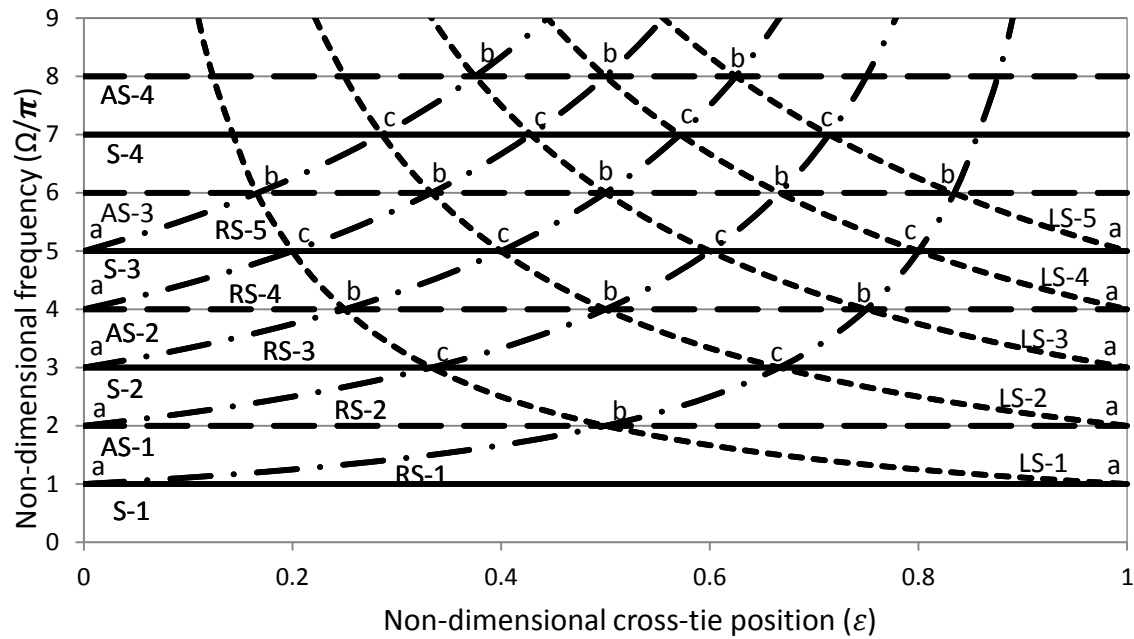


Figure 2-3. Non-dimensional modified system frequency, Ω / π , as a function of the non-dimensional cross-tie position, ε , for a twin-cable SMT network: S (global symmetric, solid line), AS (global asymmetric, broken line), RS (local, right segment, dash-dot line) and LS (local, left segment, dotted line) modes

As can be observed from the figure, while the frequencies of the global modes, which are independent of the cross-tie position, remain as constants $n\pi$ ($n=1, 2, 3 \dots$), as the cross-tie moves from the left end of the main cables towards right, the frequencies of the local LS modes gradually decrease and those of the local RS modes increase. The frequency variation of the local LS modes and RS modes are symmetric about the center line of $\varepsilon = 1/2$ and complimentary with each other. At certain cross-tie positions, more than one global or local modes have the same modal frequency. Thus, the Ω - ε curve of these modes will intersect with each other at those locations. Three types of intersections can be observed from Figure 2-3.

The first type of intersection, symbolized by “a”, represents the extreme cases of cross-tie location either at the left ($\varepsilon = 0$) or the right ($\varepsilon = 1$) support of the main cables. When $\varepsilon = 0$, the lengths of the main cable left segments are zero. Thus, the local RS modes are the same as

the global modes. Similarly, the lengths of the right cable segments become zero when $\varepsilon = 1$, and the local LS modes are the same as the global ones.

The second type of intersection, denoted by “b”, represents the coexistence of a pair of complimentary local RS mode and LS mode, along with an asymmetric global mode. The corresponding cross-tie position can be found by equating the frequency of the asymmetric global mode with one of the local modes in the complimentary pair. Use the case of the second asymmetric global mode as an example, its frequency remains as a constant when ε increases from 0 to 1, which is shown in Figure 2-3 as a horizontal broken line $\Omega=4\pi$. There are three Type II intersection points on this horizontal line. The one on the left represents the coexistence of the first LS mode, the third RS mode and the second asymmetric global mode. The frequency of the first LS mode can be computed from Eq. (2-14a) by substituting $n=1$, which gives $\Omega=\pi/\varepsilon$. By equating it with the frequency of the second asymmetric global mode, yields $\pi/\varepsilon = 4\pi$, based on which the location of the cross-tie can be determined as $\varepsilon = 1/4$. Similarly, the cross-tie locations corresponding to the other two intersection points “b” are found to be $\varepsilon = 1/2$ and $\varepsilon = 3/4$, respectively.

The third type of intersection is symbolized by “c”, it is associated with the coexistence of a complimentary pair of local RS and LS modes and a symmetric global mode. The cross-tie location where these three modes coexist can be determined by equating the frequency of the symmetric global mode with one of the local modes in the complimentary pair. For example, for the two intersection points “c” on the horizontal solid line of $\Omega=3\pi$, the corresponding cross-tie positions are $\varepsilon = 1/3$ and $\varepsilon = 2/3$, respectively.

In addition, results in Figure 2-3 clearly indicate that because of the modal property trifurcation at the intersection points “b” and “c”, the modal behavior of such a twin-cable network

is very sensitive at the vicinity of these intersections. A minor shift of the cross-tie position could lead to a drastic switch between a local mode and a global mode, or a local LS mode and the complimentary RS mode.

2.4.1.2 *Special case 1: Cross-tie at mid-span*

This is a special case of the twin-cable system discussed in Section 2.4.1.1, of which the cross-tie is placed at the mid-span. This condition yields the same segment ratio for all four main cable segments, i.e. $\varepsilon_j = 1/2$ ($j=1$ to 4) and thus $\phi_j = \Omega/2$ ($j=1$ to 4). Substitute these values into Eq. (2-13), gives

$$2\sin(\Omega)\sin\left(\frac{\Omega}{2}\right)\sin\left(\frac{\Omega}{2}\right) = 0 \quad (2-15)$$

It is worth pointing out that when studying the same type of twin-cable network, Caracoglia and Jones (2005a) derived the characteristic polynomial of the system (Eq. (2-9) in the reference), which, by setting it as zero, would give the modal frequencies of a twin-cable network with a rigid cross-tie placed at the mid-span. This characteristic polynomial, though looks slightly different in form, is actually the same as Eq. (2-15) derived independently above.

Apparently, Eq. (2-15) has three sets of solution. The first set can be derived from $\sin(\Omega) = 0$ and gives the global modes of the cable network, as discussed in Section 2.4.1.1. The second and the third set solutions to Eq. (2-15), both derived from the condition of $\sin(\Omega/2) = 0$, are the same in terms of the magnitude of the modified system frequency. However, the associated mode shapes corresponding to these two sets are totally different. The second set condition origins from $\sin(\phi_1) = \sin(\Omega/2) = 0$. It generates the modified system frequency of $\Omega = 2n\pi$ ($n = 1, 2, 3 \dots$). These are the same as the frequencies of the asymmetric global modes given by the first set. The corresponding mode shapes include vibrations of segment 1 in

main cable 1 and segment 3 in main cable 2, while segment 2 and segment 4 in these two main cables remain at rest. On the other hand, the third set condition, which is $\sin(\phi_2) = \sin(\Omega/2) = 0$, though gives the same modified system frequencies as those from the second set, represents vibrations of segment 2 in cable 1 and segment 4 in cable 2 with segment 1 and segment 3 at rest.

The mode shapes, i.e. the vectors of the normalized modal amplitudes, can be determined by substituting the frequency values into Eq. (2-10). The odd values of n generate symmetric modes, i.e. $[1 \ 1 \ 1 \ 1]^T$, while the even values of n generate three different possible vectors of normalized modal amplitudes as explained above, i.e.

- a) $[1 \ -1 \ 1 \ -1]^T$: Asymmetric global modes with in-phase motion of both cables;
- b) $[0 \ -1 \ 0 \ 1]^T$: Local modes with out-of-phase motion of right segments of the main cables;
- c) $[1 \ 0 \ -1 \ 0]^T$: Local modes with out-of-phase motion of left segments of the main cables.

Figure 2-4 portrays the first ten modes of this type of basic cable network. Modes 1 to 4 in the figure show the mode shape defined by the above four normalized modal amplitude vectors.

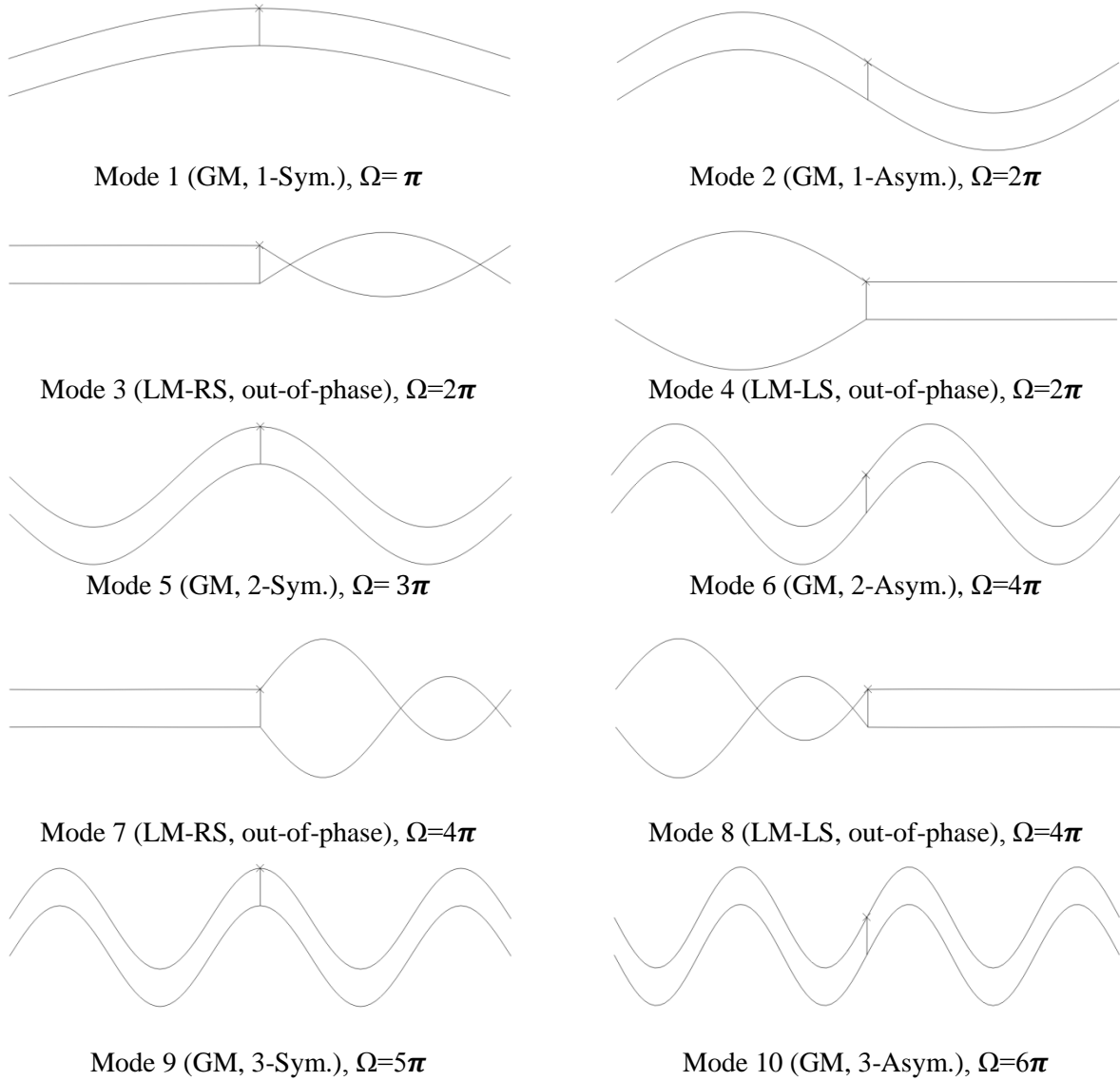


Figure 2-4 First ten modes of a symmetric twin-cable system with a rigid cross-tie at mid-span (GM: global mode, LM: local mode, Sym.: symmetric, Asym.: asymmetric, RS: right segment, LS: left segment)

2.4.1.3 *Special case 2: Cross-tie at quarter span*

In this special case of a twin-cable system, the cross-tie locates at the quarter span from the left support. This condition gives the segment ratio of the four main cable segments as $\varepsilon_1 = \varepsilon_3 = 1/4$ and $\varepsilon_2 = \varepsilon_4 = 3/4$. Substitute these values into Eq. (2-13), gives

$$2\sin(\Omega)\sin\left(\frac{\Omega}{4}\right)\sin\left(\frac{3\Omega}{4}\right) = 0$$

Again, three sets of solution are available. The first set, $\sin(\Omega) = 0$, is exactly the same as that discussed in the previous two subsections. This supports that the global modes in a SMT twin-cable network with rigid cross-tie are independent of the cross-tie position. The second and the third sets of solution are responsible for the local left-segment (LS) and the local right-segment (RS) modes respectively and these are also discussed in Section 2.4.1.1. The first ten modes are given in Figure 2-5.

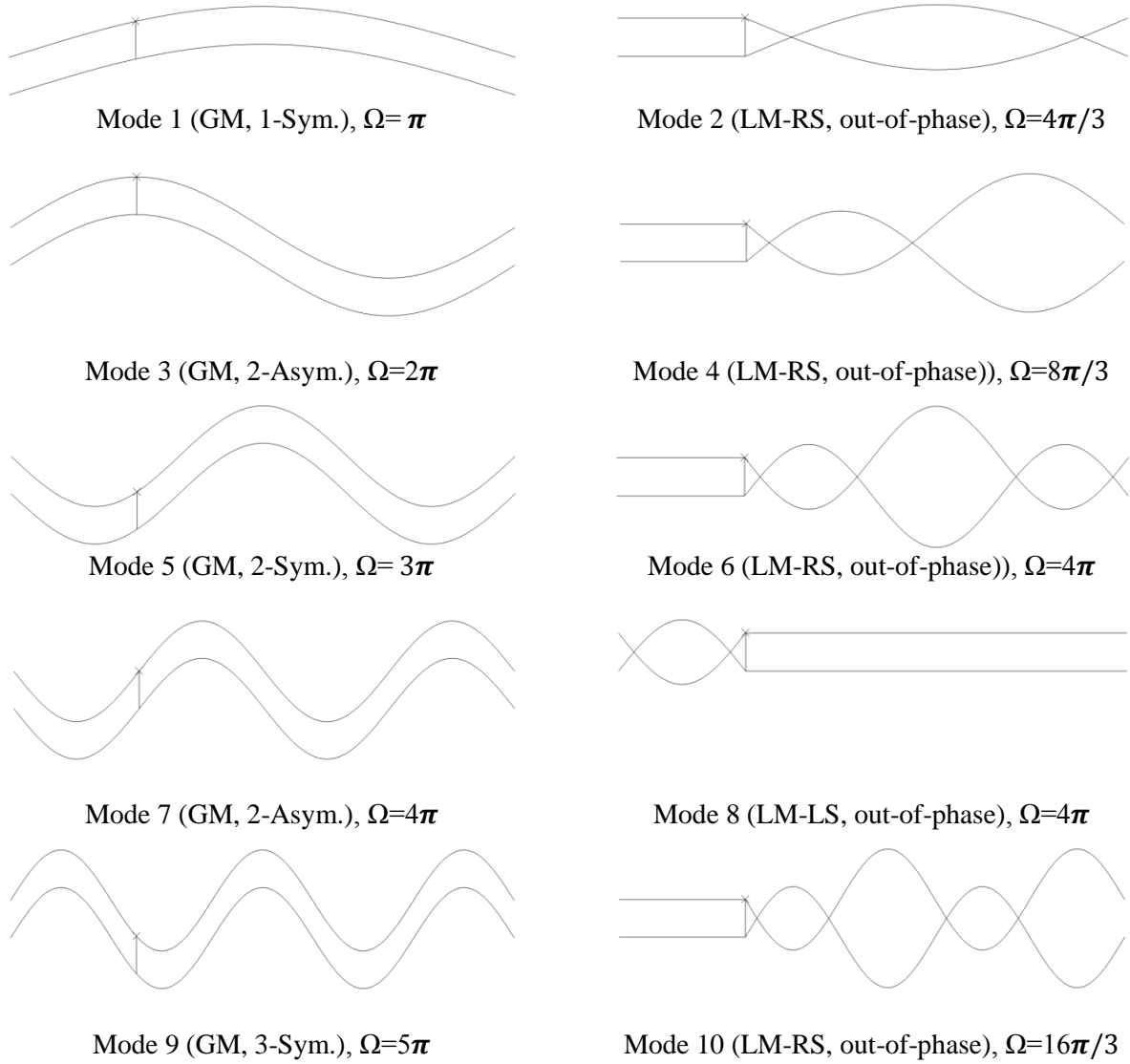


Figure 2-5 First ten modes of a symmetric twin-cable system with a rigid cross-tie at quarter span from the left end. (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 2-4)

When compared with the modal properties of a twin-cable network shown in Figure 2-4, of which the rigid cross-tie locates at the mid-span, it can be clearly observed that the global modes are independent of the cross-tie position. However, by varying $\varepsilon=1/2$ to $1/4$, the lengths of the right segments, segments 2 and 4, are increased, whereas those of the left segments, segments 2 and 4, are reduced. Consequently, the local RS modes appear early with lower frequencies and

the local LS modes are delayed due to its increased frequencies. Therefore, the modal order of the twin cable network is affected by the cross-tie position.

2.4.1.4 *Numerical example*

To validate the proposed analytical model and the derived modal solutions of a twin-cable network, numerical examples are presented in this section. The twin main cables in the example are assumed to be the same as the type AS14 cable in the Fred Hartman Bridge (Caracoglia and Jones, 2005b), i.e. both having a pretension of 1598 kN, a unit mass of 47.9 kg/m, and a length of 67.34 m. Two cross-tie locations of $\epsilon=1/2$ and $\epsilon=1/4$ are considered. The natural frequencies and the mode shapes of the first ten modes of these two basic cable networks are listed in Tables 2-1 and 2-2, respectively. The corresponding mode shapes have been portrayed in Figures 2-4 and 2-5 in the earlier discussion. A comparison between the modal analysis results in these two tables show that by moving the cross-tie from mid-span to quarter span, the modal order is changed. More local RS modes appear within the first ten modes. Such a change in the cross-tie position would increase the free length of the right segments of the main cables and makes them more flexible. Thus, the frequencies of the local RS modes would become lower, which advances their modal order. Besides, the modal analysis results yielded from the finite element simulation in ABAQUS are also presented in Tables 2-1 and 2-2. They are found to agree well with the analytical ones, which verifies the validity of the proposed analytical model.

Table 2-1: In-plane modal properties of a twin-cable network with a rigid cross-tie at mid-span

Mode number	Modal frequencies (Hz)			Mode Shapes
	Proposed Analytical Model	FEA	Error ¹ (%)	
1	1.3562	1.3563	0.007	GM, 1-Sym., in-phase
2	2.7124	2.7126	0.007	GM, 1- Asym., in-phase
3	2.7124	2.7126	0.007	LM-RS, out-of-phase
4	2.7124	2.7126	0.007	LM-LS, out-of-phase
5	4.0685	4.0676	-0.02	GM, 2-Sym., in-phase
6	5.4247	5.4226	-0.04	GM, 1- Asym., in-phase
7	5.4247	5.4226	-0.04	LM-RS, out-of-phase
8	5.4247	5.4226	-0.04	LM-LS, out-of-phase
9	6.7809	6.7749	-0.09	GM, 3-Sym., in-phase
10	8.1371	8.1273	-0.12	GM, 3- Asym., in-phase

¹ This error is calculated by taking the analytical value as the reference base.

Table 2-2: In-plane modal properties of twin-cable network with a rigid cross-tie at quarter span

Mode number	Modal frequencies (Hz)			Mode Shapes
	Proposed Analytical Model	FEA	Error ² (%)	
1	1.3562	1.3563	0.007	GM, 1-Sym., in-phase
2	1.8082	1.8084	0.01	LM-RS, out-of-phase
3	2.7124	2.7123	-0.004	GM, 1- Asym., in-phase
4	3.6165	3.6161	-0.01	LM-RS, out-of-phase
5	4.0685	4.0678	-0.02	GM, 2-Sym., in-phase
6	5.4247	5.4221	-0.05	LM-RS, out-of-phase
7	5.4247	5.4226	-0.04	GM, 1- Asym., in-phase
8	5.4247	5.4240	-0.01	LM-LS, out-of-phase
9	6.7809	6.7753	-0.08	GM, 3-Sym., in-phase
10	7.233	7.2259	-0.10	LM-RS, out-of-phase

2.4.2 Symmetric two-cable network with unequal length main cables

2.4.2.1 SMT network with cross-tie at mid-span

In a symmetric same mass-tension ratio (SMT) cable network shown in Figure 2-6, the two main cables have different lengths and the cross-tie locates at mid-span.

² This error is based at proposed analytical model value.

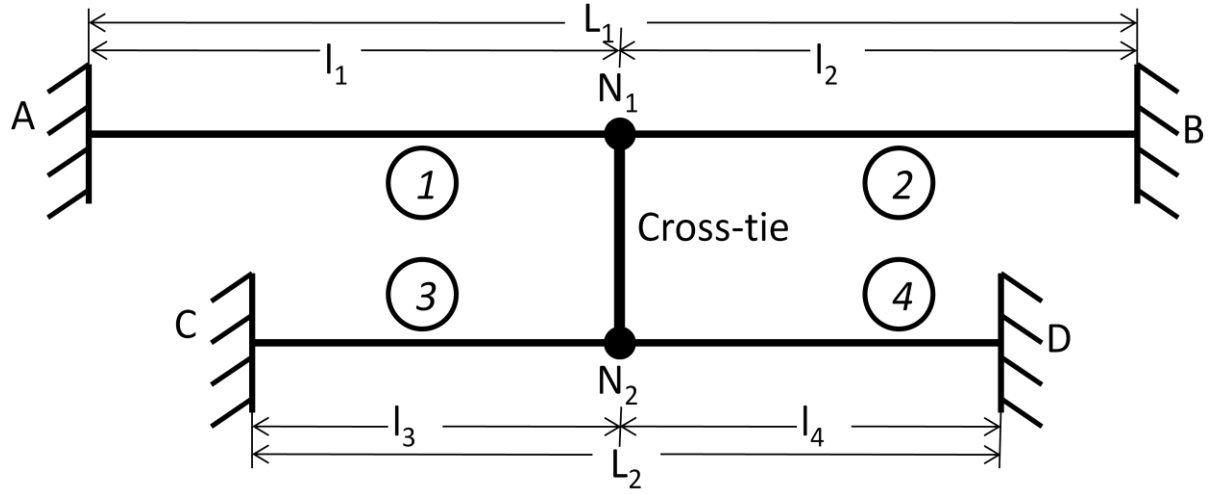


Figure 2-6 Symmetric SMT two-cable network with unequal length main cables and a rigid cross-tie at mid-span

Thus, $L_1 \neq L_2$ but $\gamma_1 = \gamma_2$ or $H_1 m_1 = H_2 m_2$. These conditions lead to the frequency ratios of $\eta_1 = 1$ and $\eta_2 = m_2 L_2 / m_1 L_1$, the mass-tension ratios of $\gamma_1 = \gamma_2 = 1$ and the same segment ratio for all segments, i.e. $\varepsilon_j = 1/2$ ($j=1$ to 4). Based on these, $\phi_1 = \phi_2 = \Omega/2$ and $\phi_3 = \phi_4 = \Omega\eta_2/2$. Substitute these values into Eq. (2-12), yields

$$\sin\left[\frac{\Omega}{2}(1 + \eta_2)\right]\sin\left(\frac{\Omega}{2}\right)\sin\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (2-16)$$

Eq. (2-16) has the following roots:

$$\Omega = 2n\pi/(1 + \eta_2) \quad (2-17a)$$

$$\Omega = 2n\pi \quad (2-17b)$$

and

$$\Omega = 2n\pi/\eta_2 \quad (2-17c)$$

where $n = 1, 2, 3 \dots$

The modified system frequencies given by Eq. (2-17a) represent the global modes, which result from the vibration coupling between main cable 1 and main cable 2. When $n=1$, this expression gives the fundamental global frequency of the cable network. Eqs. (2-17b) and (2-17c) generate the local modes of main cable 1 and main cable 2, respectively. As can be seen

from Eqs. (2-17b) and (2-17c), the local modes dominated by the longer main cable (the target cable) are the same as those of the asymmetric modes of a single main cable, while those dominated by the shorter main cable (the “colleague” cable) depend on its frequency ratio. A stiffer “colleague” cable would lead to higher frequencies of its dominated local modes. The first ten mode shapes are shown in Figure 2-7 by choosing $\eta_2 = L_2/L_1 = 0.88$, which includes both global and local modes.

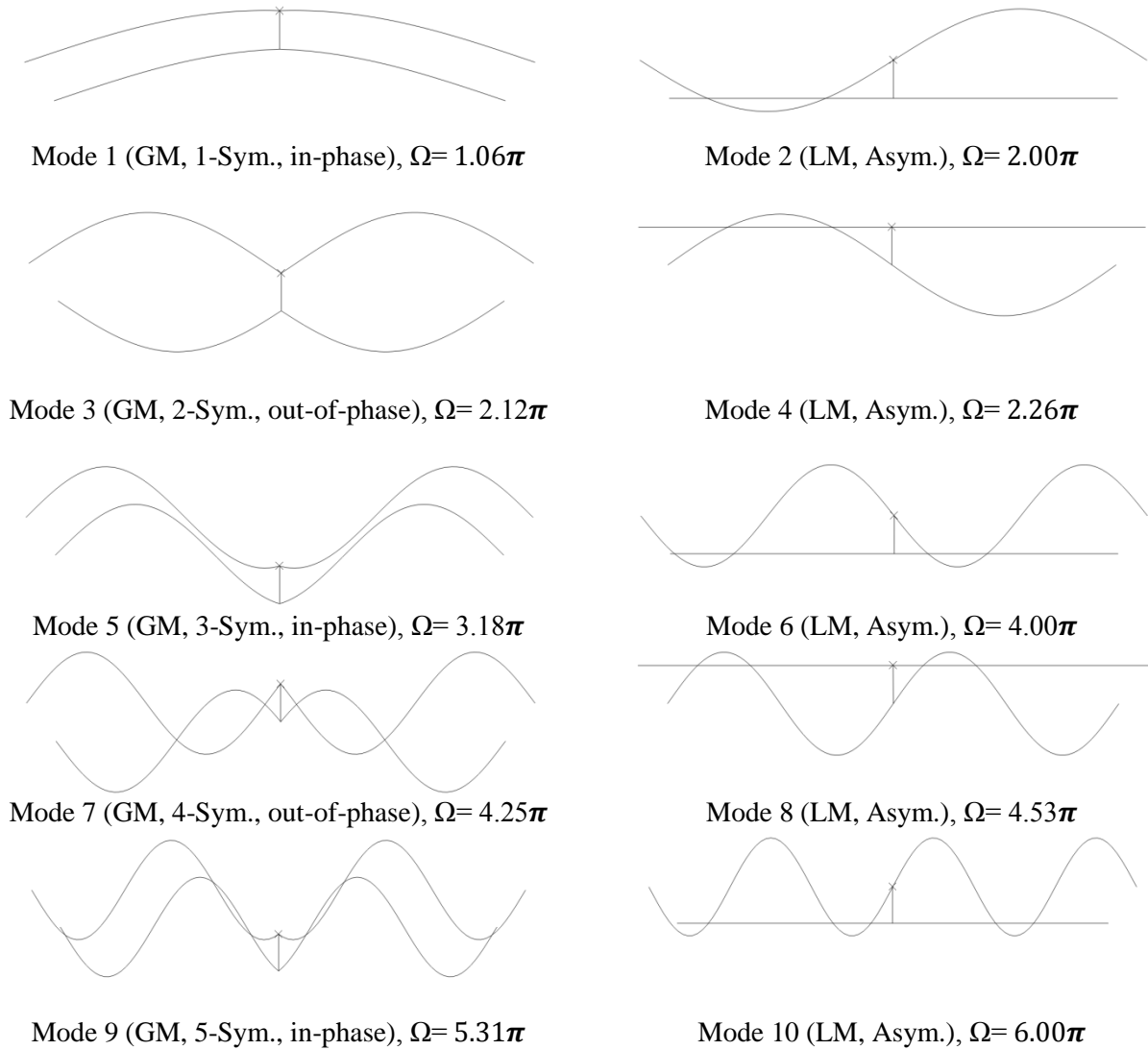


Figure 2-7 First ten modes of a symmetric SMT two-cable network with unequal length main cables and a rigid cross-tie at mid-span (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 2-4)

Further, it can be observed from Eq. (2-17a) that in this type of cable network, the frequency ratio, $\eta_2 = f_1/f_2$, of the ‘colleague cable’ would significantly affect the system frequency of the cable network. Such an impact on the fundamental frequency of the cable network is illustrated in Figure 2-8.

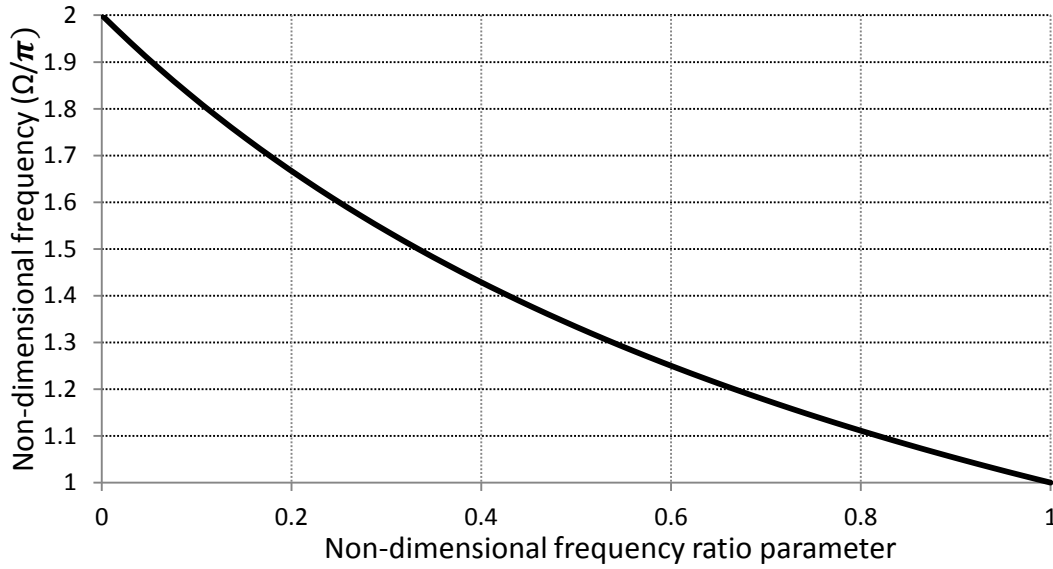


Figure 2-8 Non-dimensional modified frequency, Ω/π , as a function of frequency ratio parameter, η , for a symmetric SMT two-cable network

The pattern of the curve suggests that in a symmetric SMT two-cable-network, if the lengths of the two main cables approach to be the same, the fundamental frequency of the network becomes lower. Once the lengths are equal, the fundamental system frequency would equal to that of a single main cable. Thus, connecting a target cable to a “colleague” cable having the same mass-tension ratio and the same length with a rigid cross-tie will not help to increase its in-plane stiffness. In other words, in such a SMT network system, to further increase the in-plane stiffness of a target cable, it should be connected with a “colleague” cable of shorter length, which itself is stiffer, and has a smaller value of frequency ratio η_2 .

Numerical Example

The following cable data are used in this numerical example. The cross-tie is place at the mid-span. The modal analysis results of the first ten modes are given in Table 2-3. Some of the mode shapes are illustrated in Figure 2-7.

Main Cable 1: $H_1=1598$ kN, $m_1=47.9$ kg/m, $L_1=67.34$ m

Main Cable 2: $H_2=1598$ kN, $m_2=47.9$ kg/m, $L_2=59.52$ m

The fundamental frequency of the longer cable in this example is 1.356 Hz. As can be seen from Figure 2-7, the fundamental mode of the studied cable network is a global mode, of which the two main cables vibrate in phase with a symmetric pattern. It has a frequency of 1.440 Hz. Thus, by introducing a rigid cross-tie to connect a longer cable with its adjacent shorter neighbour, the natural frequency of the fundamental mode has been increased by 6%. The consistency between the analytical and the numerical results in Table 2-3 demonstrates again the validity of the proposed analytical model.

Table 2-3: In-plane modal properties of a symmetric SMT two-cable network with unequal cable lengths and a rigid cross-tie at mid-span

Mode number	Modal frequencies (Hz)		Mode Shapes
	Proposed Analytical Model	FEA	
1	1.4397	1.4399	GM, 1-Sym., in-phase
2	2.7124	2.7126	LM, Cable 1, 1-Asym.
3	2.8796	2.8799	GM, 2-Sym., out-of-phase
4	3.0688	3.0692	LM, Cable 2, 1-Asym.
5	4.3193	4.3185	GM, 3-Sym., in-phase
6	5.4247	5.4260	LM, Cable 1, 2-Asym.
7	5.7592	5.7570	GM, 4-Sym., out-of-phase
8	6.1374	6.1360	LM, Cable 2, 2-Asym.
9	7.1989	7.1930	GM, 5-Sym., in-phase
10	8.1371	8.1300	LM, Cable 1, 3-Asym.

2.4.2.2 DMT network with cross-tie at mid-span

For this type of configuration, the two main cables have different length and their frequency ratio and mass-tension ratio parameters are not the same. Therefore, it is termed as DMT (different mass-tension ratio) network. Since the cross-tie is placed at the mid-span, the segment ratios of all four main cable segments are the same, i.e. $\varepsilon_j = 1/2$ ($j=1$ to 4). The above conditions lead to $\phi_1 = \phi_2 = \Omega/2$ and $\phi_3 = \phi_4 = \Omega\eta_2/2$. Substitute these values into Eq. (2-12), yields

$$\left[\sin\left(\frac{\Omega}{2}\eta_2\right) \cos\left(\frac{\Omega}{2}\right) + \gamma_2 \sin\left(\frac{\Omega}{2}\right) \cos\left(\frac{\Omega}{2}\eta_2\right) \right] \sin\left(\frac{\Omega}{2}\right) \sin\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (2-18)$$

Three sets of solution are present for the system characteristic equation, Eq. (2-18), which can be derived respectively from

$$\sin\left(\frac{\Omega}{2}\eta_2\right)\cos\left(\frac{\Omega}{2}\right) + \gamma_2\sin\left(\frac{\Omega}{2}\right)\cos\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (2-19a)$$

$$\sin\left(\frac{\Omega}{2}\right) = 0 \quad (2-19b)$$

and $\sin\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (2-19c)$

The condition described by Eq. (2-19a) will give the global modes of the network, of which both main cables vibrate. The modal frequencies are the functions of the frequency ratio parameter η_2 as well as the mass-tension ratio parameter γ_2 . The second and the third sets of solution are the same as those in Section 2.4.2.1 for a SMT cable network, which represents the local modes of cable 1 and cable 2, respectively.

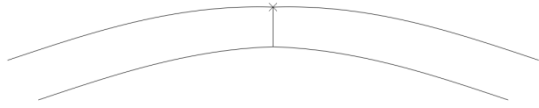
Numerical Example

The two cable types AS13 and AS14 on the Fred Hartman Bridge (Caracoglia and Jones, 2005b) are selected for the two main cables in this example. Their properties are

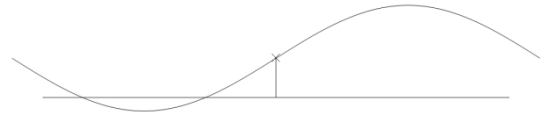
Main Cable 1: $H_1=1598$ kN, $m_1=47.9$ kg/m, $L_1=67.34$ m

Main Cable 2: $H_2=1997$ kN, $m_2=59.9$ kg/m, $L_2=59.52$ m

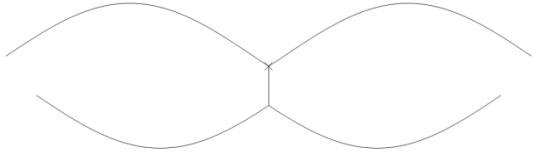
The rigid cross-tie is at the mid-point. The main cable 1 in this example is the same as that in Section 2.4.2.1. In addition, the length ratio of the two main cables, $L_2/L_1=0.88$, and the frequency ratio of main cable 2, $\eta_2=0.88$, remain the same as those of the SMT network in Section 2.4.2.1, while the mass-tension ratio of main cable 2 is increased from $\gamma_2=1$ in Section 2.4.2.1 to $\gamma_2=1.25$ here. The natural frequencies of the lowest ten modes are listed in Table 2-4, and the corresponding mode shapes are given in Figure 2-9.



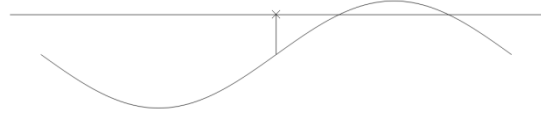
Mode 1 (GM, 1-Sym., in-phase), $\Omega = 1.07\pi$



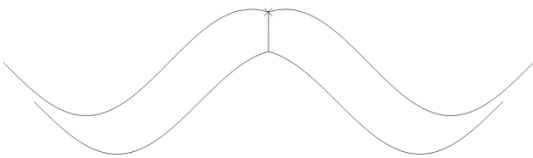
Mode 2 (LM, Asym.), $\Omega = 2.00\pi$



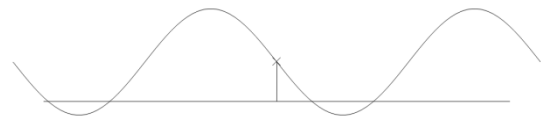
Mode 3 (GM, 2-Sym., out-of-phase), $\Omega = 2.11\pi$



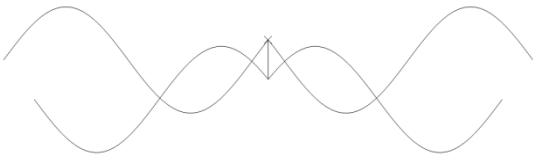
Mode 4 (LM, Asym.), $\Omega = 2.26\pi$



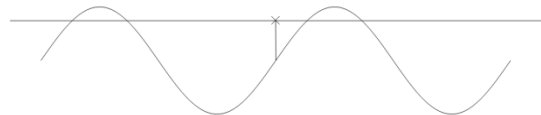
Mode 5 (GM, 3-Sym., in-phase), $\Omega = 3.21\pi$



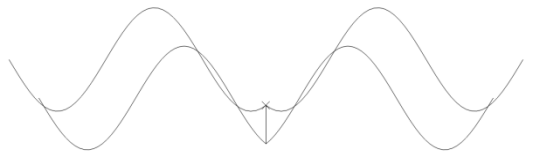
Mode 6 (LM, Asym.), $\Omega = 4.00\pi$



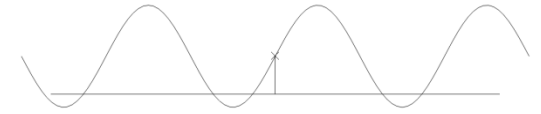
Mode 7 (GM, 4-Sym., out-of-phase), $\Omega = 4.22\pi$



Mode 8 (LM, Asym.), $\Omega = 4.53\pi$



Mode 9 (GM, 5-Sym., in-phase), $\Omega = 5.34\pi$



Mode 10 (LM, Asym.), $\Omega = 6.00\pi$

Figure 2-9 First ten modes of a symmetric DMT two-cable network with same properties as SMT network in Section 2.4.2.1 but mass-tension ratio parameter η_2 is increased by 25%. (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 2-4)

Table 2-4: In-plane modal properties of a symmetric DMT two-cable network with unequal cable lengths and a rigid cross-tie at mid-span

Mode number	Modal frequencies (Hz)		Error ³ (%)	Mode Shapes
	Proposed Analytical Model	FEA		
1	1.4496	1.4499	0.02	GM, 1-Sym., in-phase
2	2.7124	2.7126	0.007	LM, Cable 1, 1-Asym.
3	2.8605	2.8609	0.014	GM, 2-Sym., out-of-phase
4	3.0688	3.0695	0.014	LM, Cable 2, 1-Asym.
5	4.3475	4.3469	-0.014	GM, 3-Sym., out-of-phase
6	5.4247	5.4226	0.038	LM, Cable 1, 2-Asym.
7	5.7236	5.7216	-0.035	GM, 4-Sym., in-phase
8	6.1374	6.1361	-0.02	LM, Cable 2, 2-Asym.
9	7.2411	7.2355	-0.08	GM, 5-Sym., out-of-phase
10	8.1371	8.1273	-0.12	LM, Cable 1, 3-Asym.

Compared to the symmetric SMT cable network in Section 2.4.2.1, of which both cables have the same mass-tension ratio parameter, the modal order and mode shapes of the lowest ten modes remain the same in the current case. However, by connecting the same target cable (Cable 1) with its neighboring one having higher mass-tension ratio parameter, compared to the one in Section 2.4.2.1, the fundamental frequency of the current cable network is found to be slightly increased from 1.4397 Hz to 1.4496 Hz. Therefore, to enhance the in-plane stiffness of a target

³ This error is based at proposed analytical model value.

cable, it should be connected with cable(s) either having the same or higher mass-tension ratio and/or lower frequency ratio. More discussions about these two system parameters on the dynamic behavior of a cable network will be presented in a companion paper (Ahmad and Cheng, 2012). Cable 1, which is the longer cable in the network, is exactly the same as that in Section 2.4.2.1. This is the reason why mode 2 in these two different cable networks, which is the local mode of Cable 1, has the same frequency and mode shape. In the case of Cable 2, however, though the current one has higher pretension and unit mass as compared to its counterpart in Section 2.4.2.1, the frequency remains unchanged. Thus, mode 4, which is the local mode dominated by Cable 2, has exactly the same modal properties as that in the previous section.

2.4.3 Asymmetric two-cable network with a rigid cross-tie at arbitrary location

2.4.3.1 Main cables have same properties

The two-cable network shown in Figure 2-10 has an asymmetric layout.

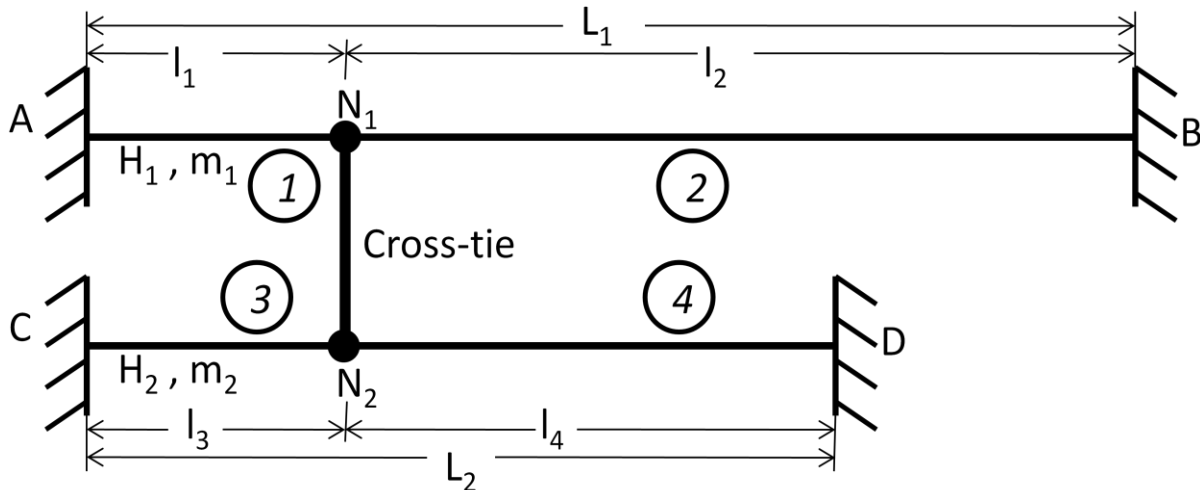


Figure 2-10 Asymmetric two-cable network with a rigid cross-tie at arbitrary location

The two main cables have the same physical properties but different lengths, i.e. $H_1 = H_2$, $m_1 = m_2$ and $L_1 \neq L_2$. These conditions give the frequency ratios of $\eta_1 = 1$ and $\eta_2 = L_2/L_1$, and the mass-tension ratios of $\gamma_1 = \gamma_2 = 1$. Since the mass-tension ratios of the two main cables are the same, it belongs to the SMT cable network. Refer to Figure 2-10, if defining the cross-tie position of the studied asymmetric two-cable network based on the length of the left segment of main cable 1, i.e. $\varepsilon = l_1/L_1$, the segment ratios of the four cable segments of the network can be expressed respectively as

$$\varepsilon_1 = \varepsilon, \quad \varepsilon_2 = 1 - \varepsilon, \quad \varepsilon_3 = \varepsilon/\eta_2, \quad \varepsilon_4 = 1 - \varepsilon/\eta_2$$

and thus

$$\phi_1 = \Omega\varepsilon, \quad \phi_2 = \Omega(1 - \varepsilon), \quad \phi_3 = \Omega\varepsilon, \quad \phi_4 = \Omega(\eta_2 - \varepsilon)$$

Substitute γ_1 , γ_2 , η_1 , η_2 , and ϕ_i ($i=1$ to 4) into Eq. (2-12), and conducting trigonometric manipulations, the characteristic equation for this type of network can be derived as

$$\{2\sin(\Omega) \sin(\Omega\eta_2) \cot(\Omega\varepsilon) - \sin[\Omega(1 + \eta_2)]\} \sin(\Omega\varepsilon) \sin(\Omega\varepsilon) = 0 \quad (2-20)$$

Three sets of solution exist for the above system characteristic equation, which are

$$2\sin(\Omega) \sin(\Omega\eta_2) \cot(\Omega\varepsilon) - \sin[\Omega(1 + \eta_2)] = 0 \quad (2-21a)$$

$$\sin(\Omega\varepsilon) = 0 \quad (2-21b)$$

$$\sin(\Omega\varepsilon) = 0 \quad (2-21c)$$

The non-dimensional system frequency Ω given by Eq. (2-21a) is a function of the cross-tie position, $\varepsilon = l_1/L_1$, and the frequency ratio η_2 of main cable 2. It is associated with the global modes of the cable network. Eqs. (2-21b) and (2-21c) generate roots representing the local modes, the frequencies of which are the functions of the cross-tie position parameter ε only, i.e. $\Omega = n\pi/\varepsilon$, where $n = 1, 2, 3, \dots$. Though these local modes have exactly the same frequency, the mode shapes associated with the local modes dominated by these two equations are quite

different. It is worth mentioning that Eq. (2-21b) is originated from the terms associated with segments 1 and 3, whereas Eq. (2-21c) from the terms associated with segments 2 and 4. Therefore, Eq. (2-21b) describes the local modes dominated by the left segments of the main cables, i.e. the LS modes, while Eq. (2-21c) gives the local modes that only involve the vibrations of the right segments, i.e. the RS modes.

Now consider an asymmetric two-cable network with $L_2=0.8L_1$, $H_1=H_2$, $m_1=m_2$ and cross-tie locates at one-fifth from the left support of the main cables. The modal frequencies of the network local modes yielded from Eqs. (2-21b) and (2-21c) would become $5n\pi$. For the same magnitude of n , the frequency $5n\pi$ could be associated with two different local modes. Figure 2-11 depicts the mode shape of the first ten modes of the studied cable network. It can be observed from the figure that mode 7 and mode 9 are both local modes which have the same frequency of $\Omega = 5\pi$, i.e. $n=1$. However, the shapes of these two modes are totally different. While mode 7 is dominated by motions of the right segments of the main cables, only the left segments in these two cables vibrate in mode 9. The same phenomenon is observed in Figure 2-4, when a twin-cable network is studied. It is worth noting that although mode 8 in Figure 2-11 also has a frequency of 5π , it is a symmetric global mode with vibrations of all four segments of the main cables. This fact implies that in the case of an asymmetric two-cable network, of which the two main cables have the same properties and the cross-tie is rigid, certain unique cross-tie position also exists which manifest tri-furcation of modal properties.

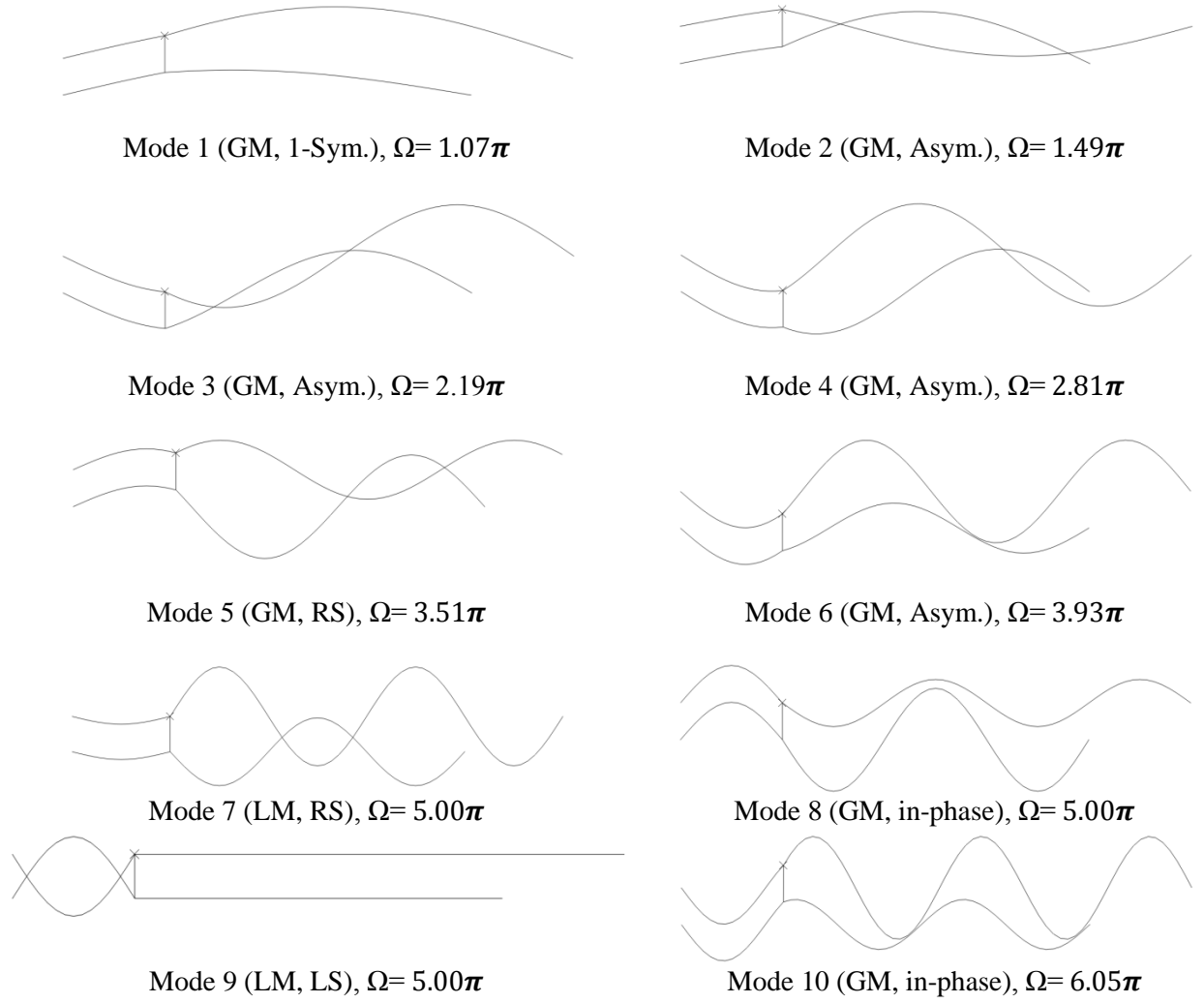


Figure 2-11 First ten modes of an asymmetric two-cable network ($H_1=H_2$, $m_1=m_2$, $L_1=0.8L_2$, rigid cross-tie locates at one-fifth from the left end)

2.4.3.2 *Main cables have different properties*

In the asymmetric DMT two-cable network studied in this section, it is assumed that the two main cables have different pretension and unit mass. By plugging these conditions into Eq. (2-12), yields the characteristic equation of the cable network as

$$\begin{aligned} & \sin(\Omega\epsilon) \sin(\Omega\eta_2\epsilon\frac{L_1}{L_2}) \{ \sin(\Omega) \sin[\Omega\eta_2(1 - \epsilon\frac{L_1}{L_2})] / \sin(\Omega\epsilon) \\ & + \gamma_2 \sin(\Omega\eta_2) \sin[\Omega(1 - \epsilon)] / \sin(\Omega\eta_2\epsilon\frac{L_1}{L_2}) \} = 0 \end{aligned} \quad (2-22)$$

Three sets of solution are present for Eq. (2-22). The first and the second set derived respectively from $\sin(\Omega\varepsilon) = 0$ and $\sin\left(\Omega\eta_2\varepsilon\frac{L_1}{L_2}\right) = 0$, represent local modes dominated by either main cable 1 or 2. These local modes are generated due to the existence of cross-tie. The position of the cross-tie, or the segment ratio, would govern whether they are lower order or higher order modes. The third set, which yielded from the summation of the terms within the main curly bracket being zero, represents the global modes of the cable network. The frequencies of these global modes are dependent of the segment ratio, the frequency ratio and the mass-tension ratio of the second main cable. Therefore, in this type of cable network, the increase of in-plane stiffness of a target cable (main cable 1) is not only affected by the properties of the connected neighboring cable, but the cross-tie position as well.

Now to evaluate the combined effect of the mass-tension ratio and the frequency ratio parameters on the in-plane modal properties of an asymmetric two-cable network, the cable network in Section 2.4.3.1 is modified by choosing $\eta_2=0.648$ and $\gamma_2=0.837$ while L_2/L_1 remains the same. It was observed that the non-dimensional fundamental frequency of the modified cable network increased slightly from 1.0687 to 1.0967. As been discussed earlier, either increase the mass-tension ratio parameter or decrease the frequency ratio of the “colleague” cable would improve the in-plane stiffness of the target cable. Compared to the asymmetric two-cable network in Section 2.4.3.1, both the mass-tension ratio and the frequency ratio of the “colleague” cable (main cable 2) in the current system have a smaller value. The former renders decrease in the system frequency, whereas the latter helps to increase the system frequency. The combined effect leads to a slight increase of the network fundamental frequency. The first ten modes derived from Eq. (2-22) are depicted in Figure 2-12.

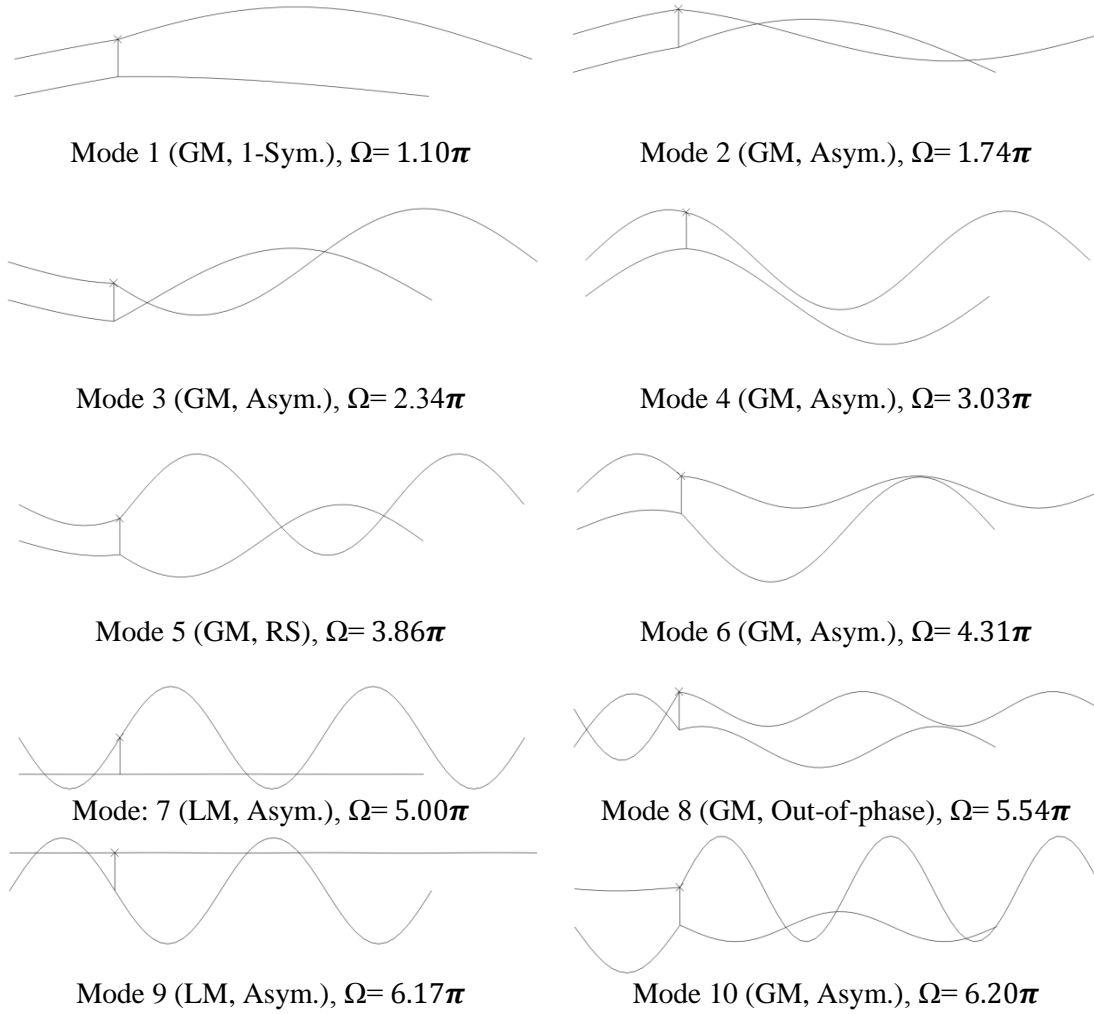


Figure 2-12 First ten modes of an asymmetric two-cable network ($H_1 \neq H_2$, $m_1 \neq m_2$, $L_1=0.8L_2$, rigid cross-tie locates at one-fifth from the left end)

2.5 CONCLUSIONS

Using cross-ties is one of the effective countermeasures to suppress unfavorable bridge stay cable vibrations. Though the strategy has been successfully applied on site, the dynamic behaviour of such a cable network is not fully understood. In this paper, an analytical model of a basic cable network consisting of two horizontally laid taut cables connected by a transverse rigid cross-tie has been proposed to study its in-plane modal behaviour. From the analytical formulation, the segment ratio, the frequency ratio, and the mass-tension ratio have been

identified as key system parameters. The proposed analytical model has been applied to basic cable networks with various configurations to demonstrate its flexibility. The modal analysis results yielded from the proposed analytical model are found to agree well with those obtained from an independent numerical simulation. The roles of various system parameters on affecting the in-plane modal behaviour of a basic cable network have been discussed. The following conclusions can be drawn from the present work:

- 1) In addition to the global modes, of which both connected main cables vibrate, local modes that only include vibrations of part of the main cables are excited due to the presence of the cross-tie.
- 2) In the case of a twin-cable network, the natural frequencies and mode shapes of the global modes are the same as those of a single main cable in the network. They are independent of the segment ratio, i.e. the cross-tie position. However, the local modes, dominated by either the right, or the left segments of the main cables, purely depend on this parameter. They form complimentary pairs. At certain cross-tie positions, the coexistence of a pair of complimentary local modes, along with either a symmetric, or an asymmetric global mode, has been observed. A minor variation of the cross-tie position at the vicinity of these segment ratio values could cause a drastic change of the modal behaviour.
- 3) Except the twin-cable system, in all the other studied basic cable network configurations, the modal frequencies of the global modes are higher than those of the target cable (the longer main cable) and are functions of different system parameters. Thus, the cross-tie solution is beneficial for enhancing the in-plane stiffness of the connected cables.
- 4) For a symmetric two-cable network with unequal length main cables and a rigid cross-tie at mid-span, the global modes depend on both the frequency ratio and the mass-tension ratio. In

a special case of the SMT network, the global modes are affected by the frequency ratio only. To further improve the in-plane stiffness of a target cable, it should be connected with neighboring cables having lower frequency ratio and/or higher mass-tension ratio. For the longer main cable, only the asymmetric local modes are excited, with the frequencies the same as those of the asymmetric modes of a single main cable; whereas the local modes of the shorter main cable depend on its frequency ratio.

- 5) The global modes of a more general asymmetric two-cable network depend on the segment ratio, the frequency ratio and the mass-tension ratio. They have higher modal frequencies when compared to those of the target cable (the longer main cable). Either one of the main cables or segments of both main cables can be excited as local modes.

It is worth pointing out that using cross-tie(s) to connect a problematic cable with its neighbor(s) would not only affect its modal behaviour, but will have impact on its damping property as well. The latter will be further explored in the future publications. To have a more comprehensive understanding of the in-plane modal behaviour of a typical cable network, the analytical formulation developed for a basic cable network in the current study will be extended to a general cable network in a companion paper (Ahmad and Cheng, 2012), which consists of n taut main cables connected transversely by a single line of rigid cross-ties. The roles of different system parameters on the dynamic behaviour of a cable network will be further discussed.

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CHAPTER 3

Analytical Study on In-plane Modal Behavior of Stay Cables Connected by Cross-ties: Part II: General Cable Network

3.1 Introduction

In recent years, cable-stayed bridges are designed and built with much longer span length. The Sutong Bridge over the Yangtze River in China is the longest cable-stayed bridge in the world at present. It has a center span length of 1088 m and set a new record at the time of completion (Zhang et al, 2009). As a result, the length of the stay cables becomes progressively longer, which renders them more vulnerable to dynamic excitations. Much effort has been made in developing and evaluating effective vibration control strategies to suppress or eliminate unfavourable cable vibrations.

Different solutions, such as applying surface modification (Matsumoto et al, 1992; Zhan et al, 2008), installing external dampers (Pacheco et al, 1993; Krenk, 2000; Tabatabai and Mehrabi, 2000; Main and Jones, 2002; Fujino and Hoang, 2008; Cheng et al, 2010) and cross-ties (Gimsing, 1993; Yamaguchi and Nagahawatta, 1995; Virlogeux, 1998; Caracoglia and Jones, 2005b), have been used on site with various degree of effectiveness. Though external dampers are widely used in the field, the application of cross-ties is becoming more popular.

To develop more effective cross-tie solutions, a number of studies have been conducted to investigate the dynamic behaviour and mechanism of a cable network (Ehsan and Scanlan, 1989; Yamaguchi and Nagahawatta, 1995; Caracoglia and Jones, 2005a; Bosch and Park, 2005; Sun et al, 2007; Caracoglia and Zuo, 2009; Zhou et al, 2011). A similar effort has been made by the authors in a companion paper (Ahmad and Cheng, 2012), of which an analytical model of a basic cable network consisting of two horizontally laid main cables and a transverse vertical cross-tie was proposed. Though such a cable network has been studied by different researchers,

the dynamic behaviour of a more general cable network, of which n horizontally laid main cables are connected transversely by cross-tie(s), has rarely seen in the literatures. In addition, it is also noticed that in majority of the existing works, the key system parameters of a cable network have not been clearly identified. Thus, the effect of different system parameters on the network behaviour is still not clear.

Viewing these needs, in the present paper, the analytical model and the approach proposed for studying the in-plane modal behaviour of a basic cable network in the companion paper (Ahmad and Cheng, 2012) will be extended to analyze the in-plane free vibration of a general cable network. An analytical model and modal solutions of an extended cable network consisting of four horizontally laid taut cables connected by a single line of rigid transverse cross-ties will be developed first based on the basic cable network model formulation. It will then be further extended to developing an analytical model of a general cable network where n main cables are present in the system. To verify the validity of the proposed analytical model, it will be applied to the simplest scenario of a basic cable network by setting $n=2$, and comparing the modal analysis results with those obtained from the basic cable network model proposed in the companion paper. Then, the proposed analytical model will be applied to general cable networks of different configurations, and the results will be compared with those yielded from independent numerical simulations. The key system parameters of a typical cable network will be identified from the proposed analytical model. The role of each identified system parameter on the in-plane modal behaviour of a cable network will be extensively examined.

3.2 In-plane Free Vibration of a General Cable Network

3.2.1 Description of the system

Before formulating the system equation of a general cable network, an extended cable network with four horizontally laid main cables will be studied first.

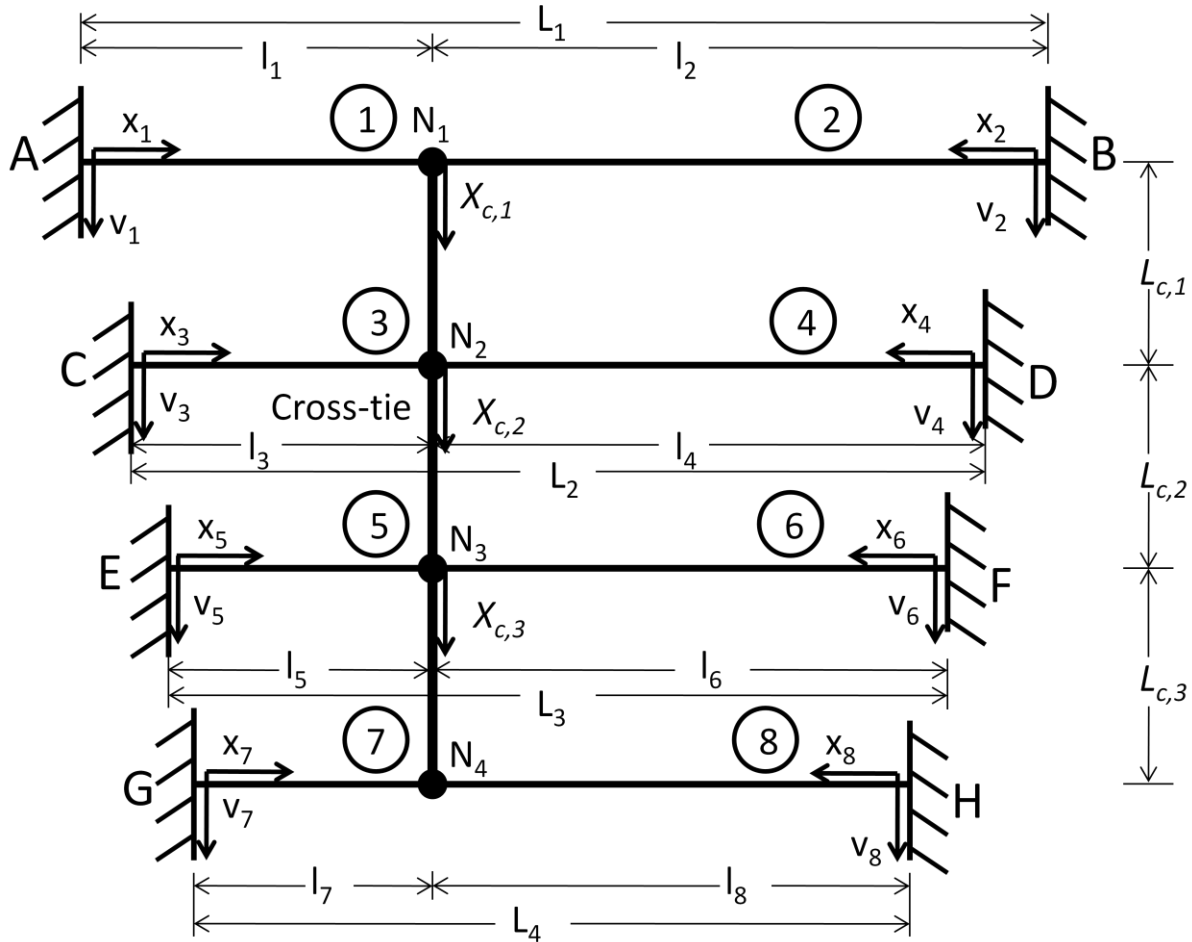


Figure 3-1 Schematic diagram of the mathematical model of an extended cable network

As portrayed in Figure 3-1, this extended model comprises of four main cables of different lengths, with L_1 being the length of the longest cable in the network. To control vibrations of main cable 1, it is connected to three other main cables through a single line of transverse rigid cross-ties of length $L_{c,i}$ ($i=1, 2, 3$), which divide each main cable into two segments. They are labelled as shown in Figure 3-1. Assume the mass per unit length of cable i is

m_i and the pre-stressing force is H_i ($i=1, 2, 3, 4$). The position of the cross-tie is l_1 from the left support of main cable 1. The transverse displacement of the main cables and the axial displacement of the cross-ties are considered positive downward and negative upward. All four main cables are assumed to be fixed at both ends.

3.2.2 Formulation of the system equation

All the assumptions made for developing the analytical model of an extended cable network remain the same as those for a basic cable network, i.e. a) The main cables are idealized as taut cables; b) The additional dynamic tensions in the main cables are neglected; c) The main cables are fixed at both ends; d) The main cables have the in-plane transverse vibration as their dominant motion, the longitudinal vibration is neglected; e) The cross-ties have the longitudinal vibration as their dominant motion, the transverse motion is neglected; f) The cross-ties are rigid. As explained in the companion paper (Ahmad and Cheng, 2012), the last assumption is made not only to simplify the formulation and solution for a general cable network, but considers the fact that the fundamental frequency of a cable network will only be slightly decreased when choosing flexible type of cross-tie as oppose to rigid type used on real bridges (Caracoglia and Jones, 2005b). Based on the equation of motion describing the transverse and axial vibrations of a single cable (Ahmad and Cheng, 2012), those of the main cables and the cross-ties in the proposed analytical model can be expressed as

$$\bar{v}_{2i-1}(x_{2i-1}) = B_{2i-1} \sin(\Omega \eta_i x_{2i-1} / L_i) \quad i=1 \text{ to } 4 \quad (3-1a)$$

$$\bar{v}_{2i}(x_{2i}) = B_{2i} \sin(\Omega \eta_i x_{2i} / L_i) \quad i=1 \text{ to } 4 \quad (3-1b)$$

$$\bar{u}_{c,k}(x_{c,k}) = C_k \cos(\Omega \eta_{c,k} x_{c,k} / L_{c,k}) + D_k \sin(\Omega \eta_{c,k} x_{c,k} / L_{c,k}) \quad k=1 \text{ to } 3 \quad (3-1c)$$

where $\Omega = \pi f / f_1$ is the non-dimensional frequency of the modified cable network with f being the frequency of the modified cable network and f_1 the fundamental frequency of the target cable

(main cable 1) in the network; $\eta_i=f_1/f_i$ and $\eta_{c,k}=f_1/f_{c,k}$ are the frequency ratios, where f_i and $f_{c,k}$ are the natural frequency of the i^{th} cable and the k^{th} cross-tie, respectively; L_i is the length of the i^{th} cable in the network; B_{2i-1} and B_{2i} ($i=1$ to 4) are the shape function constants of the eight main cable segments and C_k and D_k ($k=1$ to 3) are the shape function constants of rigid cross-ties, as shown in Figure 3-1.

The following boundary, compatibility and equilibrium conditions are applied to this extended cable network model:

Boundary conditions

$$v_{2i-1}(0, t) = 0, \quad v_{2i}(0, t) = 0 \quad i=1, 2, 3, 4 \quad (3-2a)$$

Compatibility conditions

$$v_{2i-1}(l_{2i-1}, t) = v_{2i}(l_{2i}, t) \quad i=1, 2, 3, 4 \quad (3-2b)$$

$$v_{2i-1}(l_{2i-1}, t) = v_{2i-3}(l_{2i-3}, t) \quad i=2, 3, 4 \quad (3-2c)$$

Equilibrium conditions

$$\sum_{i=1}^4 \left(\frac{\partial v_{2i-1}}{\partial x_{2i-1}} \Big|_{x_{2i-1}=l_{2i-1}} + \frac{\partial v_{2i}}{\partial x_{2i}} \Big|_{x_{2i}=l_{2i}} \right) H_i = 0 \quad (3-2d)$$

Eq. (3-2d) represents the longitudinal equilibrium of the single line of cross-ties by isolating them from the network. The summation of the transverse components of the main cable tension due to vibration should be zero at any arbitrary time instant t .

Applying the above conditions to Eq. (3-1) and expressing the resulting equations into a matrix form, yields

$$[R]\{X\}=\{0\} \quad (3-3)$$

where

$$[R] = \begin{bmatrix} \sin(\phi_1) & -\sin(\phi_2) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & -\sin(\phi_4) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin(\phi_5) & -\sin(\phi_6) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sin(\phi_7) & -\sin(\phi_8) \\ \sin(\phi_1) & 0 & -\sin(\phi_3) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & 0 & -\sin(\phi_5) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin(\phi_5) & 0 & \sin(\phi_7) & 0 \\ \gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4) & \gamma_3 \cos(\phi_5) & \gamma_3 \cos(\phi_6) & \gamma_4 \cos(\phi_7) & \gamma_4 \cos(\phi_8) \end{bmatrix}$$

is the coefficient matrix and $\{X\} = [B_1 \ B_2 \ B_3 \ B_4 \ B_5 \ B_6 \ B_7 \ B_8]^T$ is the vector containing all the unknown shape function constants of the main cable segments, and $\{0\}$ is the null vector. In the coefficient matrix $[R]$, ϕ in its general form, for the first segment of each main cable, is $\phi_{2i-1} = \Omega \eta_i \varepsilon_{2i-1}$, where $\varepsilon_{2i-1} = l_{2i-1}/L_i$ is the segment ratio of the first segment of the i^{th} main cable ($i=1$ to 4); whereas for the second segment of each main cable, $\phi_{2i} = \Omega \eta_i \varepsilon_{2i}$, where $\varepsilon_{2i} = l_{2i}/L_i$ is the segment ratio of the second segment of the i^{th} main cable ($i=1$ to 4); $\eta_i = f_1/f_i$ and $\gamma_i = \sqrt{H_i m_i / H_1 m_1}$ are respectively the frequency ratio and the mass-tension ratio parameter of the i^{th} main cable; f_i , m_i , H_i are respectively the fundamental frequency, the unit mass and the pretension of the i^{th} cable in the network ($i=1$ to 4).

The infinite sets of non-trivial solution to Eq. (3-3) can be achieved by setting the determinant of the coefficient matrix $[R]$ to zero, i.e.

$$\begin{vmatrix} \sin(\phi_1) & -\sin(\phi_2) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & -\sin(\phi_4) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin(\phi_5) & -\sin(\phi_6) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sin(\phi_7) & -\sin(\phi_8) \\ \sin(\phi_1) & 0 & -\sin(\phi_3) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & 0 & -\sin(\phi_5) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin(\phi_5) & 0 & \sin(\phi_7) & 0 \\ \gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4) & \gamma_3 \cos(\phi_5) & \gamma_3 \cos(\phi_6) & \gamma_4 \cos(\phi_7) & \gamma_4 \cos(\phi_8) \end{vmatrix} = 0$$

After expanding the above equation and making all the trigonometric simplifications, the following system characteristic equation can be obtained, i.e.

$$\gamma_1 \sin(\Omega \eta_1) \sin(\phi_3) \sin(\phi_4) \sin(\phi_5) \sin(\phi_6) \sin(\phi_7) \sin(\phi_8)$$

$$\begin{aligned}
& + \gamma_2 \sin(\Omega \eta_2) \sin(\phi_1) \sin(\phi_2) \sin(\phi_5) \sin(\phi_6) \sin(\phi_7) \sin(\phi_8) \\
& + \gamma_3 \sin(\Omega \eta_3) \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \sin(\phi_4) \sin(\phi_7) \sin(\phi_8) \\
& + \gamma_4 \sin(\Omega \eta_4) \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \sin(\phi_4) \sin(\phi_5) \sin(\phi_6) = 0 \quad (3-4)
\end{aligned}$$

In this equation, there are four main terms and each corresponds to one main cable in the network. Every term is the product of the mass-tension ratio parameter of one main cable, the combined Sine term of that main cable and the Sine terms of both segments of the remaining three main cables. Based on this observation, the frequency equation of a more general cable network consisting of n horizontally laid main cables and a single line of rigid cross-ties, as shown in Figure 3-2, can be written following the same pattern as

$$\begin{aligned}
& \gamma_1 \sin(\Omega \eta_1) \sin(\phi_3) \sin(\phi_4) \sin(\phi_5) \sin(\phi_6) \dots \sin(\phi_{2n-1}) \sin(\phi_{2n}) \\
& + \gamma_2 \sin(\Omega \eta_2) \sin(\phi_1) \sin(\phi_2) \sin(\phi_5) \sin(\phi_6) \dots \sin(\phi_{2n-1}) \sin(\phi_{2n}) \\
& + \gamma_3 \sin(\Omega \eta_3) \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \sin(\phi_4) \dots \sin(\phi_{2n-1}) \sin(\phi_{2n}) + \dots \\
& + \gamma_n \sin(\Omega \eta_n) \sin(\phi_1) \sin(\phi_2) \sin(\phi_3) \sin(\phi_4) \sin(\phi_5) \dots \sin(\phi_{2n-3}) \sin(\phi_{2n-2}) = 0 \quad (3-5)
\end{aligned}$$

where Ω is the modified system frequency of the cable network and n is the number of main cables in the cable network.

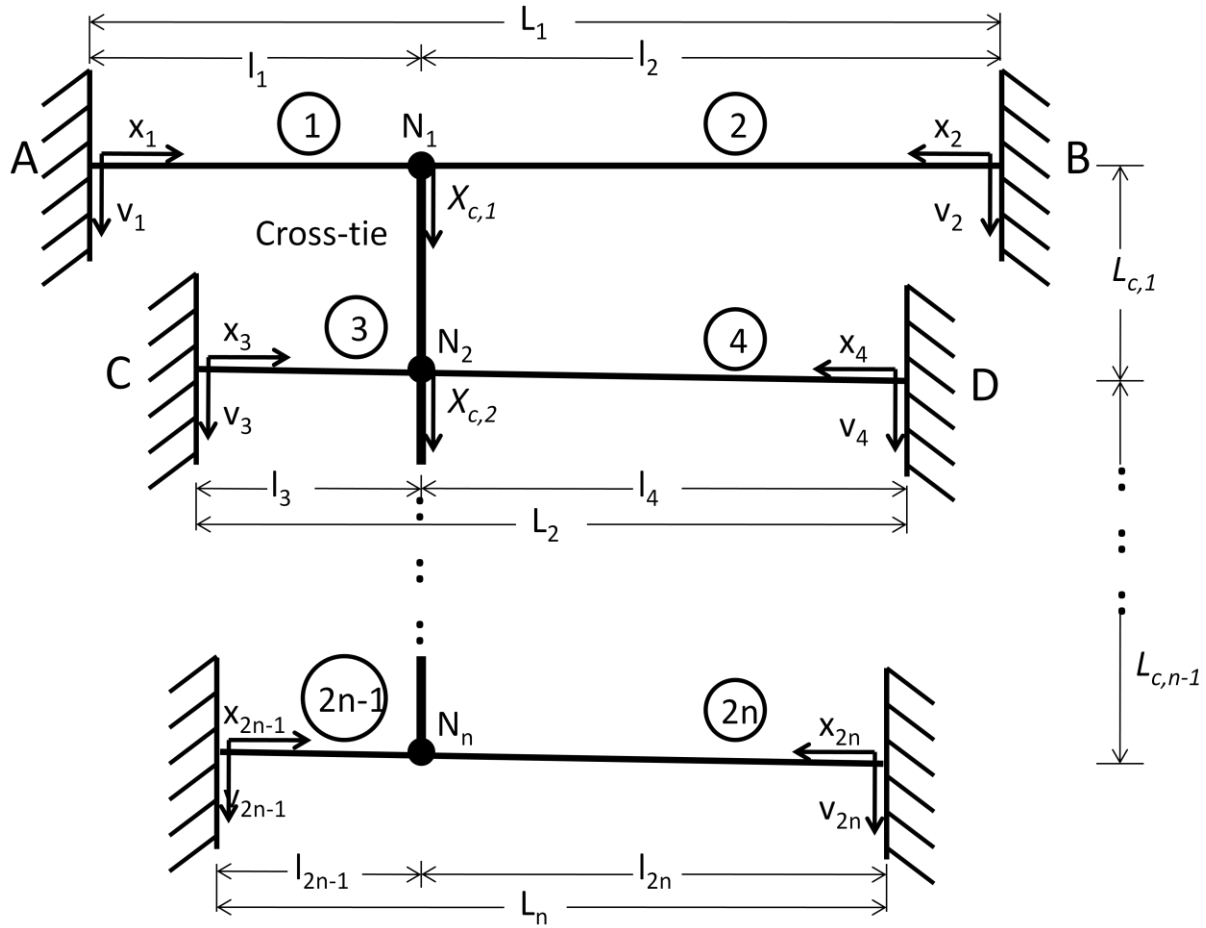


Figure 3-2 Schematic diagram of the mathematical model of a general cable network

3.3 Applications to General Cable Networks with Different Configurations

In this section, the proposed analytical model of a general cable network will be applied to cable network systems with different configurations and cable properties. As a model validation, a finite element model of a general cable network is developed in ABAQUS 6.9. The B21 beam element is selected to simulate the behaviour of the main cables, whereas the RB2D2 rigid body element is chosen for the rigid cross-tie (Ahmad and Cheng, 2012). The results obtained from the proposed analytical model of a general cable network will be compared with those from the numerical simulations.

3.3.1 Symmetric SMT two-cable network with a single line of rigid cross-ties at mid-span

A SMT network is a cable network of which all the main cables have the same mass-tension ratio parameter, i.e. $\gamma_i = 1 (i = 1 \text{ to } n)$. Three SMT cable networks, all with symmetric layout, are shown in Figure 3-3. In all three cable networks, the target cable (main cable 1) remains the same and the rigid cross-tie locates at the mid-span. The pretension, unit mass and length of the cables are:

Network-1:	$H_1=1598 \text{ kN}$	$m_1=48 \text{ kg/m}$	$L_1=72 \text{ m}$
	$H_2=1918 \text{ kN}$	$m_2=40 \text{ kg/m}$	$L_2=72 \text{ m}$
Network -2:	$H_1=1598 \text{ kN}$	$m_1=48 \text{ kg/m}$	$L_1=72 \text{ m}$
	$H_2=1598 \text{ kN}$	$m_2=48 \text{ kg/m}$	$L_2=60 \text{ m}$
Network -3:	$H_1=1598 \text{ kN}$	$m_1=48 \text{ kg/m}$	$L_1=72 \text{ m}$
	$H_2=1278 \text{ kN}$	$m_2=60 \text{ kg/m}$	$L_2=48 \text{ m}$

Though from Figure 3-3, the three cable networks look quite different and the main cables seem to have different physical and mechanical properties, the key system parameters such as the frequency ratio, the mass-tension ratio and the segment ratio remain the same, i.e. for all three cable networks in Figure 3-3, we have

Frequency ratio: $\eta_1=1, \eta_2=0.833$

Mass-tension ratio: $\gamma_1=\gamma_2=1.0$

Segment ratio: $\varepsilon_j=1/2 (j=1 \text{ to } 4)$

Substitute these values into Eq. (3-5), yields

$$\sin \left[\frac{\Omega}{2} (\eta_1 + \eta_2) \right] \sin \left(\frac{\Omega}{2} \eta_1 \right) \sin \left(\frac{\Omega}{2} \eta_2 \right) = 0 \quad (3-6a)$$

or
$$\sin \left[\frac{\Omega}{2} (1 + \eta_2) \right] \sin \left(\frac{\Omega}{2} \right) \sin \left(\frac{\Omega}{2} \eta_2 \right) = 0 \quad (3-6b)$$

Eq. (3-6) has the following roots:

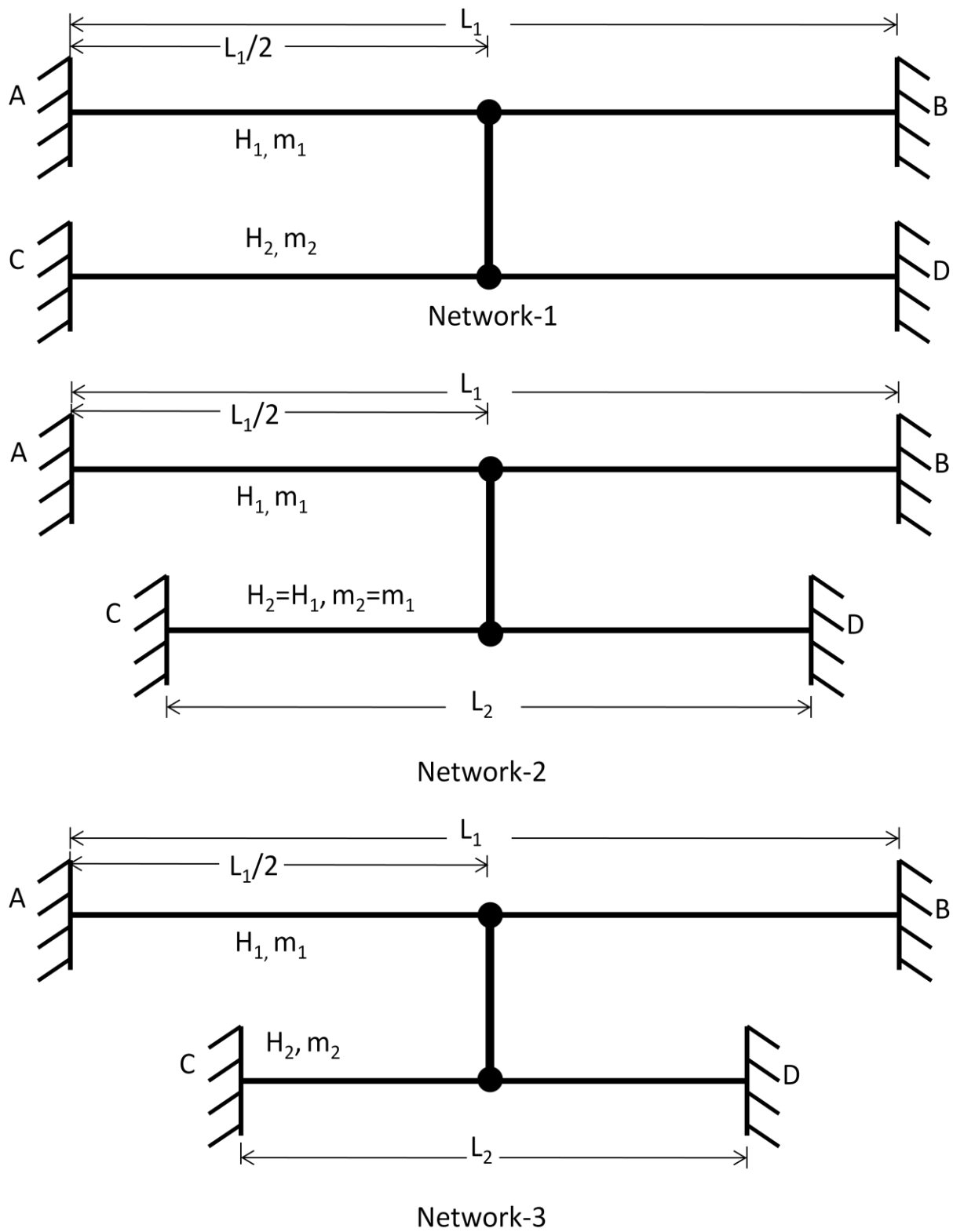


Figure 3-3 Schematic diagrams of three SMT cable networks having the same system parameters

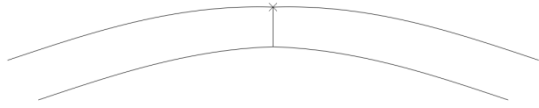
$$\Omega = 2n\pi/(1 + \eta_2) \quad (3-7a)$$

$$\Omega = 2n\pi \quad (3-7b)$$

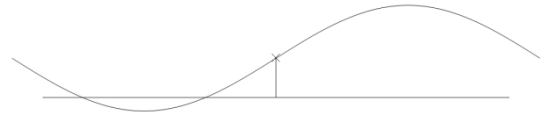
and $\Omega = 2n\pi/\eta_2 \quad (3-7c)$

where $n = 1, 2, 3 \dots$. As been discussed in the companion paper (Ahmad and Cheng, 2012), Eq. (3-7a) gives the modal frequency of the network global modes, whereas Eqs. (3-7b) and (3-7c) represent the local modes dominated by either the longer or the shorter main cables, respectively. While the local modes of the longer cable have the same frequencies as the asymmetric modes of a single cable, the frequencies of the local modes of the shorter cable depend on its frequency ratio η_2 . As discussed earlier, the in-plane modal behaviour of a cable network is dictated by the identified system parameters η , γ and ε . As far as these dimensionless parameters are the same, the modal properties in terms of the modal frequencies, the mode shapes and the modal order should be the same. The modal analysis results of these three symmetric SMT cable networks, which have the same system parameters, are listed in Table 3-1.

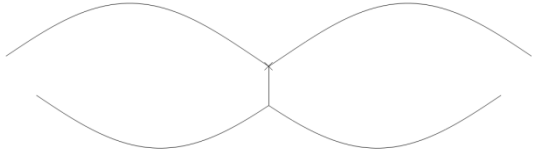
The fundamental frequency of main cable 1 in all three cable networks is 1.267 Hz. As can be seen from Table 3-1, the fundamental mode of the studied cable network has a frequency of 1.382 Hz. Thus, by introducing a rigid cross-tie to connect a target cable (main cable 1 in the example) with its adjacent neighbour having higher frequency, $\eta_2=f_1/f_2=0.833$, the fundamental frequency of the target cable has been increased by 9%. The modal analysis results yielded from an independent finite element simulation are also given in Table 3-1. The consistency between the analytical and the numerical results demonstrates the validity of the proposed analytical model. As an example, the mode shapes of the first ten modes of cable Network 2 are illustrated in Figure 3-4.



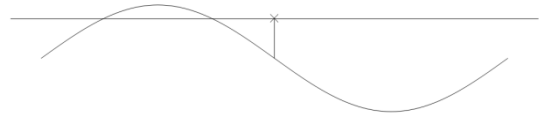
Mode 1 (GM, 1-Sym., in-phase), $\Omega = 1.09\pi$



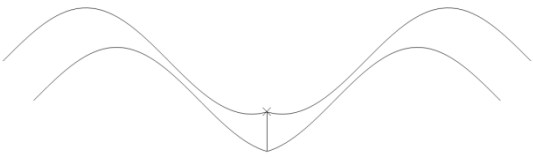
Mode 2 (LM, Asym.), $\Omega = 2.00\pi$



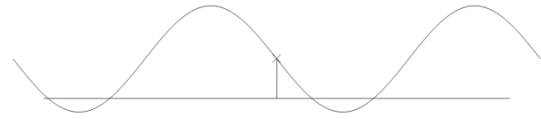
Mode 3 (GM, 2-Sym., out-of-phase), $\Omega = 2.18\pi$



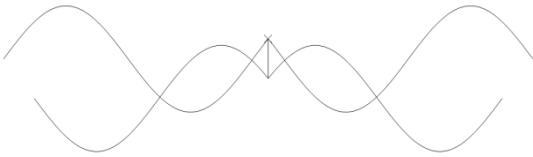
Mode 4 (LM, Asym.), $\Omega = 2.40\pi$



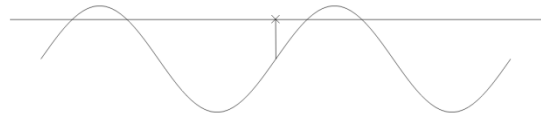
Mode 5 (GM, 3-Sym., in-phase), $\Omega = 3.27\pi$



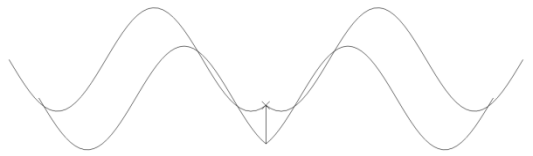
Mode 6 (LM, Asym.), $\Omega = 4.00\pi$



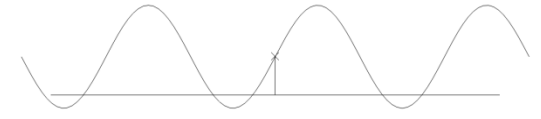
Mode 7 (GM, 4-Sym., out-of-phase), $\Omega = 4.36\pi$



Mode 8 (LM, Asym.), $\Omega = 4.80\pi$



Mode 9 (GM, 5-Sym., in-phase), $\Omega = 5.45\pi$



Mode 10 (LM, Asym.), $\Omega = 6.00\pi$

Figure 3-4 First ten modes of a symmetric SMT two-cable network (network-2) with system parameters as frequency ratio $\eta_2=0.833$ and segment ratios $\varepsilon_j=1/2$ ($j=1, 2, 3, 4$) (GM: global mode, LM: local mode, Sym.: symmetric, Asym.: asymmetric)

Table 3-1: In-plane modal properties of a symmetric SMT two-cable network with system parameters as frequency ratio $\eta_2=0.833$ and segment ratio $\epsilon_j=1/2$ ($j=1$ to 4)

Mode number	Modal frequency (Hz)		Error ⁴ (%)	Mode Shape
	Proposed Analytical Model	FEA		
1	1.3823	1.3823	0.00	GM, 1-Sym., in-phase
2	2.5342	2.5342	0.00	LM, Cable 1, 1-Asym.
3	2.7645	2.7640	-0.02	GM, 2-Sym., out-of-phase
4	3.0410	3.0405	-0.02	LM, Cable 2, 1-Asym.
5	4.1468	4.1443	-0.06	GM, 3-Sym., in-phase
6	5.0683	5.0628	-0.11	LM, Cable 1, 2-Asym.
7	5.5290	5.5224	-0.12	GM, 4-Sym., out-of-phase
8	6.0820	6.0752	-0.11	LM, Cable 2, 2-Asym.
9	6.9113	6.8986	-0.18	GM, 5-Sym., in-phase
10	7.6025	7.5822	-0.26	LM, Cable 1, 3-Asym.

3.3.2 Symmetric SMT four-cable network with a single line of rigid cross-ties at mid-span

In this cable network example, the four stay cables are selected in such a way that the mass-tension ratio parameters of all the main cables are the same. Therefore, it becomes a SMT cable network. These cables are connected through a single line of rigid cross-ties at mid-span and the configuration is similar to the cable network in Figure 3-1. The main cables 2, 3 and 4 have the same frequency, but different from that of the target cable (main cable 1). Thus, the

⁴ This error is based at proposed analytical model value.

frequency ratios are $\eta_1 \neq \eta_2 = \eta_3 = \eta_4$, the mass-tension ratios are $\gamma_i = 1 (i = 1, 2, 3, 4)$, and all cable segments have the same segment ratio of $\varepsilon_j = 1/2 (j=1 \text{ to } 8)$. Substitute these values into Eq. (3-5), yields

$$\begin{aligned} & \sin(\Omega\eta_1) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \\ & + \sin(\Omega\eta_2) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \\ & + \sin(\Omega\eta_3) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \\ & + \sin(\Omega\eta_4) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_3\right) = 0 \end{aligned}$$

After trigonometric simplifications, the above equation can be written as

$$\begin{aligned} & 2\sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \left[\sin\frac{\Omega}{2}(\eta_1 + \eta_2) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \right. \\ & \quad \left. + \sin\frac{\Omega}{2}(\eta_3 + \eta_4) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \right] = 0 \end{aligned} \quad (3-8)$$

Denoting $\eta_{2/3} = \eta_2 = \eta_3$, and $\eta_{3/4} = \eta_3 = \eta_4$, the summation of the two terms within the square brackets of Eq. (3-8) can be further expressed as

$$\begin{aligned} & \sin\frac{\Omega}{2}(\eta_1 + \eta_2) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) + \sin\frac{\Omega}{2}(\eta_3 + \eta_4) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \\ & = \sin\left(\frac{\Omega}{2}\eta_{2/3}\right) \sin\left(\frac{\Omega}{2}\eta_{3/4}\right) \left[\sin\frac{\Omega}{2}(\eta_1 + \eta_2) + 2\cos\left(\frac{\Omega}{2}\eta_{3/4}\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \right] \end{aligned}$$

Therefore, Eq. (3-8) becomes

$$\begin{aligned} & 2\sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_{2/3}\right) \sin\left(\frac{\Omega}{2}\eta_{3/4}\right) \left[\sin\frac{\Omega}{2}(\eta_1 + \eta_2) \right. \\ & \quad \left. + 2\cos\left(\frac{\Omega}{2}\eta_{3/4}\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \right] = 0 \end{aligned}$$

Apparently, the above equation has seven sets of roots, i.e.

$$\sin\left(\frac{\Omega}{2}\eta_1\right) = 0 \quad (3-9a)$$

$$\sin\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (3-9b)$$

$$\sin\left(\frac{\Omega}{2}\eta_3\right) = 0 \quad (3-9c)$$

$$\sin\left(\frac{\Omega}{2}\eta_4\right) = 0 \quad (3-9d)$$

$$\sin\left(\frac{\Omega}{2}\eta_{2/3}\right) = 0 \quad (3-9e)$$

$$\sin\left(\frac{\Omega}{2}\eta_{3/4}\right) = 0 \quad (3-9f)$$

$$\sin\frac{\Omega}{2}(\eta_1 + \eta_2) + 2\sin\left(\frac{\Omega}{2}\eta_1\right)\cos\left(\frac{\Omega}{2}\eta_{3/4}\right) = 0 \quad (3-9g)$$

Eqs. (3-9a) to (3-9d) represent respectively the local modes of main cables 1 to 4, of which only a single main cable vibrates, and the other three cables in the network remain at rest. Eqs. (3-9e) and (3-9f) also give local modes of the cable network. However, the former is associated with the modes of which main cables 2 and 3 dominate the vibration, whereas the latter corresponds to those where oscillations of main cables 3 and 4 take the domination. Noticing that in the studied cable network, main cables 2, 3 and 4 have the same frequency ratio, and also the definition of $\eta_{2/3}$ and $\eta_{3/4}$, we have $\eta_2 = \eta_3 = \eta_4 = \eta_{2/3} = \eta_{3/4}$. Therefore, Eqs. (3-9b) to (3-9f) will yield the same modal frequency, but associated with five different local modes. They are modes 4 to 8 depicted in Figure 3-5, by assuming $\eta_1 = 1$ and $\eta_2 = \eta_3 = \eta_4 = 0.833$.

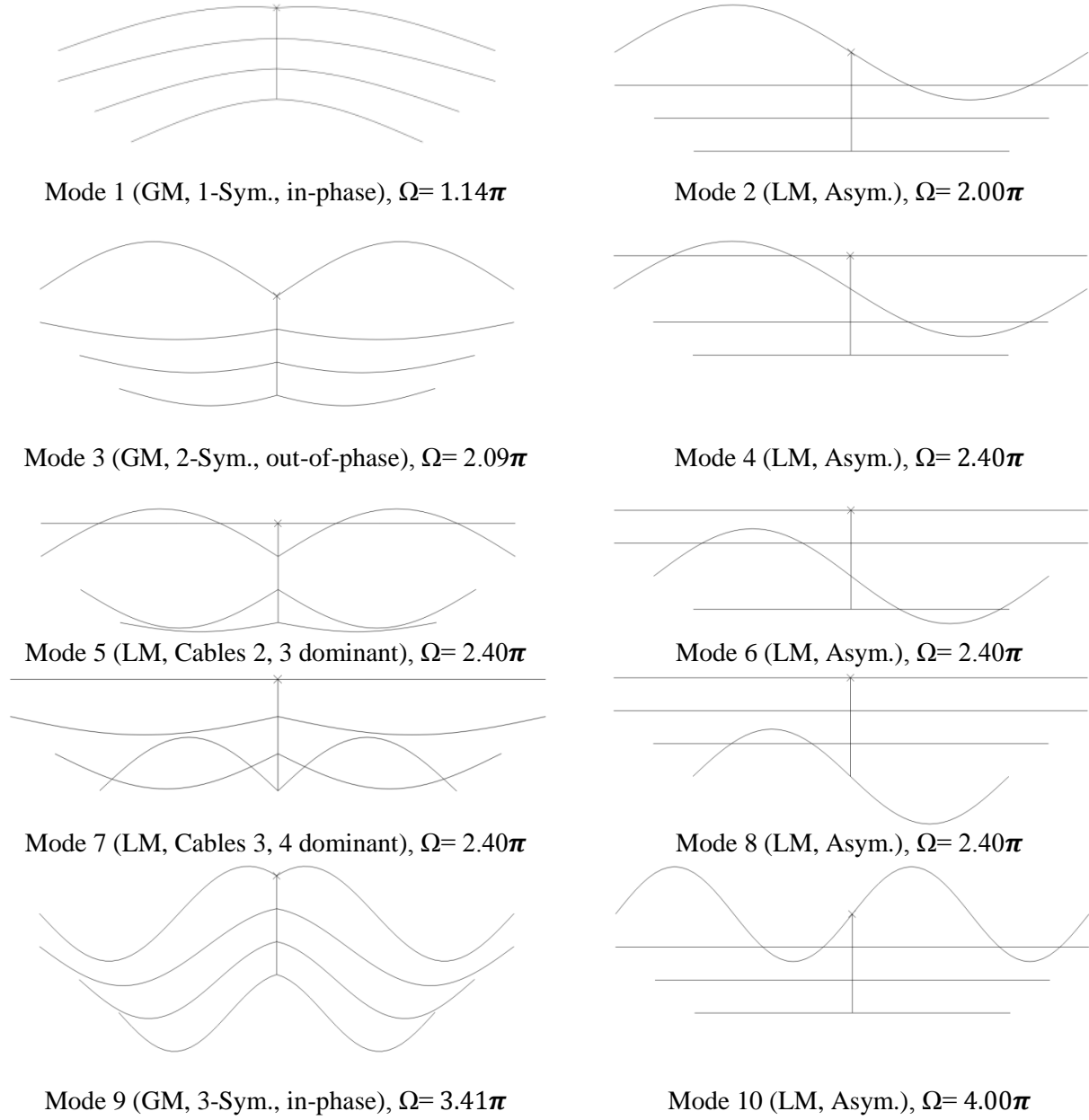


Figure 3-5 First ten modes of a symmetric SMT four-cable network with system parameters as frequency ratio $\eta_i=0.833$ ($i=2, 3, 4$) and segment ratios $\varepsilon_j=1/2$ ($j=1$ to 8) (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 3-4)

Eq. (3-9g) generates the global modes. It has the form of summation of two terms. The first term, $\sin \Omega(\eta_1 + \eta_2)/2$, which is related to the main cables 1 and 2, is the same as the first Sine term in Eq. (3-6a), it gives the frequency of the global modes of a SMT two-cable network. The second term, $2 \sin(\Omega\eta_1/2) \cos(\Omega\eta_{3/4}/2)$, actually contains the contributions from main

cables 1, 3 and 4. The $\sin(\Omega\eta_1/2)$ term is associated with main cable 1, and each of the two $\cos(\Omega\eta_{3/4}/2)$ terms is actually associated with either main cable 3 or main cable 4. The second term in Eq. (3-9g) is responsible for the change in the fundamental frequency of the cable network due to connecting to more neighbouring cables. Thus, in the case of a general cable network with N main cables and $\eta_2 = \eta_3 = \dots = \eta_N$, cables 3 to N will all contribute to the second term in Eq. (3-9g), i.e. Eq. (3-9g) will become

$$\sin\frac{\Omega}{2}(\eta_1 + \eta_2) + (N - 2)\sin\left(\frac{\Omega}{2}\eta_1\right)\cos\left(\frac{\Omega}{2}\eta_{3/N}\right) = 0 \quad (3-9g')$$

where $\eta_{3/N} = \eta_3 = \eta_4 = \dots = \eta_N$.

3.3.3 Symmetric DMT four-cable network with a single line of rigid cross-ties at mid-span

The four stay cables selected in this case are the same as those discussed in Section 3.3.2, except with the frequency ratio and the mass-tension ratio parameters of cables 2, 3 and 4 different from those of cable 1, i.e. $\eta_1 \neq \eta_2 = \eta_3 = \eta_4$ and $\gamma_1 \neq \gamma_2 = \gamma_3 = \gamma_4$. Therefore, it becomes a DMT cable network. Again, these cables are connected through a single line of rigid cross-ties at mid-span and laid symmetrically similar as the one in Figure 3-1. All eight cable segments in the network have the same segment ratio, i.e. $\varepsilon_j=1/2$ ($j=1$ to 8). Substitute these values into Eq. (3-5), yields

$$\begin{aligned} & \gamma_1 \sin(\Omega\eta_1) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \\ & + \gamma_2 \sin(\Omega\eta_2) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \\ & + \gamma_3 \sin(\Omega\eta_3) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \sin\left(\frac{\Omega}{2}\eta_4\right) \\ & + \gamma_4 \sin(\Omega\eta_4) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_3\right) \sin\left(\frac{\Omega}{2}\eta_3\right) = 0 \end{aligned}$$

After trigonometric simplifications and also introducing the same notations $\eta_{2/3}$ and $\eta_{3/4}$ as in the previous section, i.e. $\eta_{2/3} = \eta_2 = \eta_3$ and $\eta_{3/4} = \eta_3 = \eta_4$, the above system characteristic equation of a symmetric DMT four-cable network can be expressed as

$$2\sin\left(\frac{\Omega}{2}\eta_1\right)\sin\left(\frac{\Omega}{2}\eta_2\right)\sin\left(\frac{\Omega}{2}\eta_3\right)\sin\left(\frac{\Omega}{2}\eta_4\right)\sin\left(\frac{\Omega}{2}\eta_{2/3}\right)\sin\left(\frac{\Omega}{2}\eta_{3/4}\right)\left\{\left[\gamma_1\cos\left(\frac{\Omega}{2}\eta_1\right)\sin\left(\frac{\Omega}{2}\eta_2\right)+\gamma_2\cos\left(\frac{\Omega}{2}\eta_2\right)\sin\left(\frac{\Omega}{2}\eta_1\right)\right]+\sin\left(\frac{\Omega}{2}\eta_1\right)\left[\gamma_3\cos\left(\frac{\Omega}{2}\eta_3\right)+\gamma_4\cos\left(\frac{\Omega}{2}\eta_4\right)\right]\right\}=0 \quad (3-10)$$

Seven sets of roots can be found for Eq. (3-10), which yields respectively from

$$\sin\left(\frac{\Omega}{2}\eta_1\right)=0 \quad (3-11a)$$

$$\sin\left(\frac{\Omega}{2}\eta_2\right)=0 \quad (3-11b)$$

$$\sin\left(\frac{\Omega}{2}\eta_3\right)=0 \quad (3-11c)$$

$$\sin\left(\frac{\Omega}{2}\eta_4\right)=0 \quad (3-11d)$$

$$\sin\left(\frac{\Omega}{2}\eta_{2/3}\right)=0 \quad (3-11e)$$

$$\sin\left(\frac{\Omega}{2}\eta_{3/4}\right)=0 \quad (3-11f)$$

$$\left[\gamma_1\cos\left(\frac{\Omega}{2}\eta_1\right)\sin\left(\frac{\Omega}{2}\eta_2\right)+\gamma_2\cos\left(\frac{\Omega}{2}\eta_2\right)\sin\left(\frac{\Omega}{2}\eta_1\right)\right]+\sin\left(\frac{\Omega}{2}\eta_1\right)\left[\gamma_3\cos\left(\frac{\Omega}{2}\eta_3\right)+\gamma_4\cos\left(\frac{\Omega}{2}\eta_4\right)\right]=0 \quad (3-11g)$$

Similar as the SMT four-cable network discussed in the previous section, Eqs. (3-11a) to (3-11f) give the modal frequencies associated with the local modes, with Eqs. (3-11a) to (3-11d) represent respectively the local modes solely involves the vibration of each of the main cables 1 to 4, and Eqs. (3-11e) and (3-11f) are associated with the local modes dominated by the motions of either main cables 2 and 3 or main cables 3 and 4, respectively. Eq. (3-11g) gives the modal frequencies of the network global modes. Though it seems to look quite different from Eq. (3-9g), which is responsible for the global modes of a SMT four-cable network, a closer examination of the form of Eq. (3-11g) reveals that the difference is purely caused by the

condition of $\gamma_1 \neq \gamma_2 = \gamma_3 = \gamma_4$, i.e. the mass-tension ratios of main cables 2, 3 and 4 are not the same as that of cable 1 in the current case. Overall, it can still be considered that the L.H.S. of Eq. (3-11g) contains two main terms. The first term, enclosed by the first square bracket, is only related to main cables 1 and 2; whereas the second term physically represents the contributions of main cables 3 and 4 to the change of the network in-plane stiffness. If there are N cables in the network, then main cables 3 to N will all contribute to this second term, and Eq. (3-11g) becomes

$$\left[\gamma_1 \cos\left(\frac{\Omega}{2}\eta_1\right) \sin\left(\frac{\Omega}{2}\eta_2\right) + \gamma_2 \cos\left(\frac{\Omega}{2}\eta_2\right) \sin\left(\frac{\Omega}{2}\eta_1\right) \right] + \sum_{i=3}^N \gamma_i \sin\left(\frac{\Omega}{2}\eta_1\right) \cos\left(\frac{\Omega}{2}\eta_i\right) \quad (3-11g')$$

It is interesting to note that both the modal frequencies and the mode shapes of the six sets of local modes determined from Eqs. (3-11a) to (3-11f) remain the same as those of the SMT network in the previous section. This implies that the local modes in a DMT cable network will not be affected by the mass-tension ratio of the main cables. However, this system parameter will influence the modal properties of the global modes as is evident from Eq. (3-11g). Since the mass-tension ratio parameter γ_i ($i=1$ to 4) appears as factors of the harmonic terms in Eq. (3-11g), its role is to modify the modal frequencies rather than the shapes of global modes, if a corresponding SMT cable network is taken as the base for comparison.

Figure 3-6 illustrates the mode shape of the first ten modes of a symmetric DMT four-cable network having the same geometric layout, frequency ratios and segment ratios as the one in Section 3.3.1, except a 50% increase for the mass-tension ratio parameters of main cables 2, 3 and 4, i.e.

Frequency ratio: $\eta_1 = 1, \eta_2 = \eta_3 = \eta_4 = 0.833$

Mass-tension ratio: $\gamma_1 = 1, \gamma_2 = \gamma_3 = \gamma_4 = 1.5$

Segment ratio: $\varepsilon_j = 1/2$ ($j=1$ to 8)

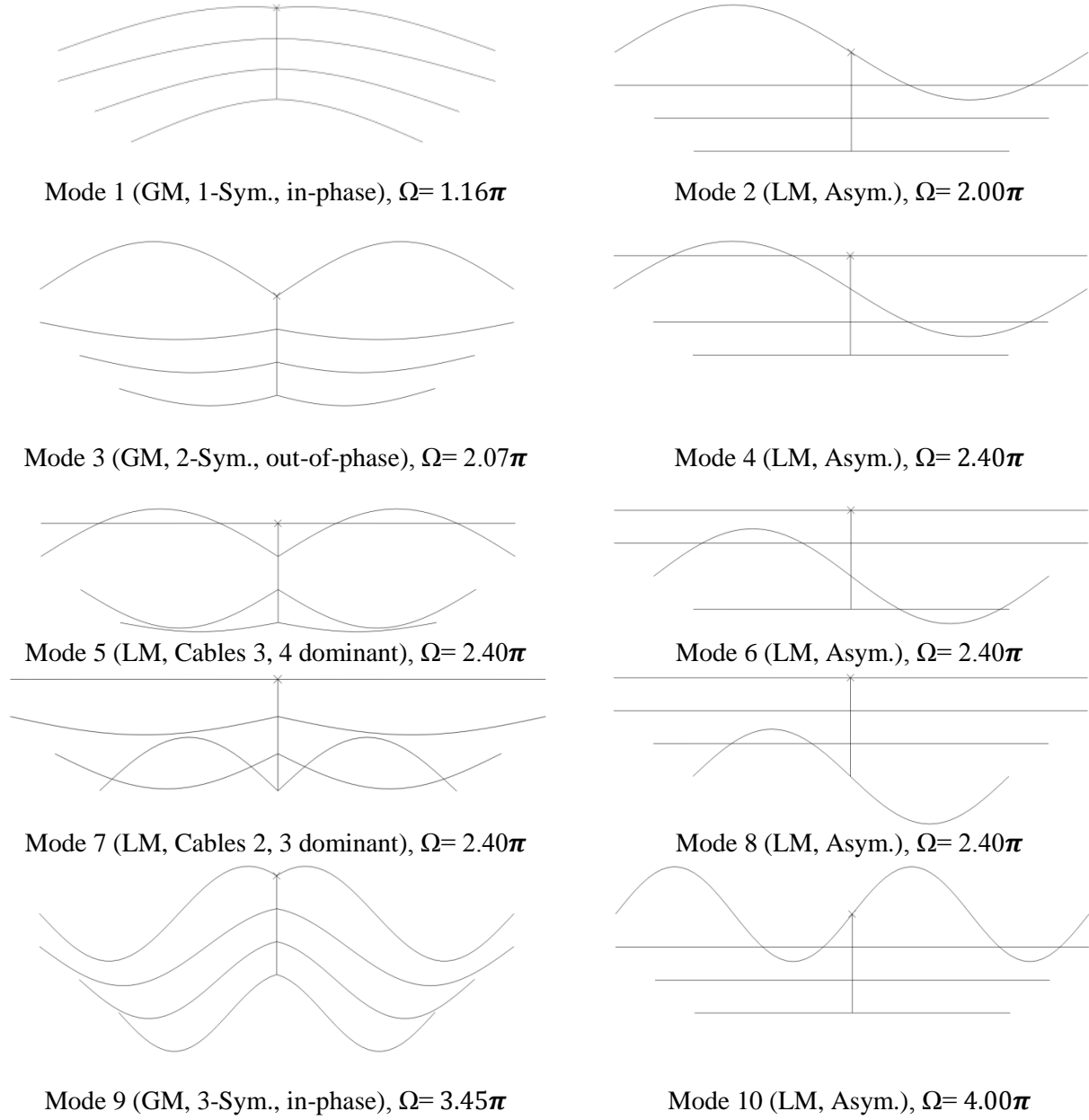


Figure 3-6 First ten modes of a symmetric DMT four-cable network with system parameters as frequency ratio $\eta_i=0.833$ ($i=2, 3, 4$) mass-tension ratio $\gamma_i=1.50$ ($i=2, 3, 4$) and segment ratio $\varepsilon_j=1/2$ ($j=1$ to 8) (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 3-4)

A comparison between Figures 3-5 and 3-6 reveals that changing the mass-tension ratio parameter of the neighbouring cables alone will not affect the modal order and mode shapes of the cable network. Although the frequencies of the global modes may be affected, those of the local modes remain the same.

An independent numerical simulation is conducted to analyze the in-plane modal behaviour of the DMT cable network of interest. This set of numerical results, as well as the analytical set obtained from the proposed general cable network, are listed in Table 3-2. For comparison purpose, the modal analysis results of the SMT network discussed in the previous section are also given in the table.

Table 3-2: In-plane modal properties of a symmetric SMT and a symmetric DMT four-cable network ($\eta_i=0.833$, $i=2, 3, 4$) with a single line of rigid cross-ties at mid-span ($\varepsilon_j=1/2$, $j=1$ to 8)

Mode number	Modal frequency (Hz)				Mode Shape
	SMT network		DMT network		
	Proposed Analytical Model	FEA	Proposed Analytical Model	FEA	
1	1.4473	1.4473	1.4663	1.4663	GM, 1-Sym., in-phase
2	2.5342	2.5338	2.5342	2.5338	LM, Cable 1, 1-Asym.
3	2.6487	2.6483	2.6177	2.6172	GM, 2-Sym., out-of-phase
4	3.0410	3.0398	3.0410	3.0398	LM, Cable 2, 1-Asym.
5	3.0410	3.0399	3.0410	3.0399	LM, Cable 2, 3 dominant
6	3.0410	3.0402	3.0410	3.0402	LM, Cable 3, 1-Asym.
7	3.0410	3.0403	3.0410	3.0403	LM, Cable 3, 4 dominant
8	3.0410	3.045	3.0410	3.045	LM, Cable 4, 1-Asym.
9	4.3229	4.3191	4.3773	4.3733	GM, 3-Sym., in-phase
10	5.0683	5.0628	5.0683	5.0628	LM, Cable 1, 2-Asym.

As explained earlier, the only difference between the two cable networks in Table 3-2 is that in the DMT network, the mass-tension ratio parameters of the three neighbouring cables, i.e. main cables 2, 3 and 4, are 50% higher than those in the SMT network. It can be clearly seen from the table that the two networks have the same modal order. While the modal frequencies of the three global modes (modes 1, 3, 9) are affected by the mass-tension ratio parameter, those of the other seven local modes are independent of this system parameter. In terms of the system

fundamental frequency, a 1.3% increment, from 1.4473 Hz in the SMT network to 1.4663 Hz in the DMT network, is observed. This suggests that a 50% increase in the mass-tension ratio parameter of the neighbouring cable(s) will have very limited contribution to the in-plane stiffness of a cable network. The mode shapes of the first ten modes remain the same. These results are consistent with the conclusions obtained by comparing the analytical solutions of the modal properties given by Eqs. (3-9) and (3-11). In addition, the modal analysis results yielded from the proposed analytical model are found to agree well with those by the numerical simulations. This proves again the validity of the proposed general cable network analytical model.

3.4 Parametric Study

To identify the key system parameters in a cable network which would affect its in-plane dynamic response, the characteristic equation of a general cable network, Eq. (3-5), is revisited. Consider the simplest configuration of a two-cable network with symmetric layout, as shown in Figure 3-7, and assume the rigid cross-tie locates at an arbitrary location l_1 from the left end of main cable 1.

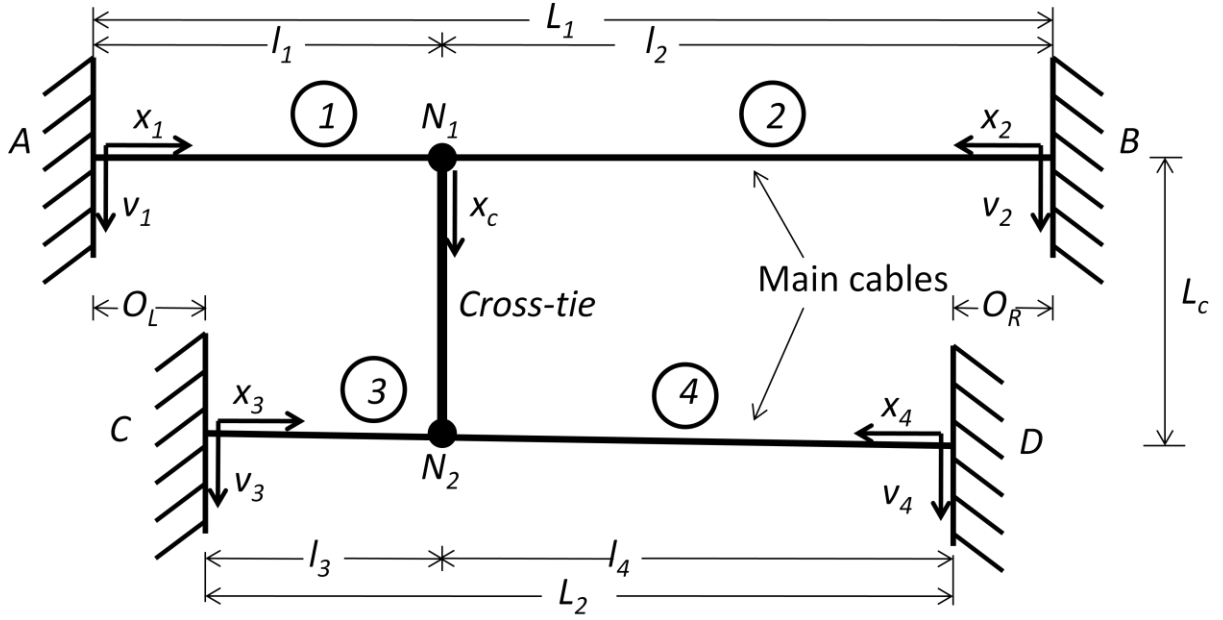


Figure 3-7 Schematic diagram of the mathematical model of a two-cable network

If define the segment ratio of the cable segments based on that of cable segment 1 of the first main cable, and introduce $\varepsilon = \varepsilon_1 = l_1/L_1$, the segment ratio of the other three cable segments in the network can be expressed as

$$\varepsilon_2 = 1 - \varepsilon_1 = 1 - \varepsilon$$

$$\begin{aligned} \varepsilon_3 &= [l_1 - (L_1 - L_2)/2]/L_2 \\ &= 1/2 + (L_1/L_2)(\varepsilon - 1/2) \\ &= 1/2 + \lambda_2(\varepsilon - 1/2) \end{aligned}$$

and

$$\varepsilon_4 = 1 - \varepsilon_3 = 1/2 - \lambda_2(\varepsilon - 1/2)$$

where $\lambda_2 = L_1/L_2$ is the newly identified system parameter defined as the ratio between the length of main cable 1 and that of main cable 2. It will be termed as “length ratio” in the subsequent discussions. To be more general, if a cable network has more than two main cables, the length ratio of the i^{th} cable will be defined as $\lambda_i = L_1/L_i$, where L_i is the length of the i^{th} cable.

The characteristic equation of the current two-cable network can be obtained by reducing Eq. (3-5) to

$$\gamma_1 \sin(\Omega\eta_1) \sin(\phi_3)\sin(\phi_4) + \gamma_2 \sin(\Omega\eta_2) \sin(\phi_1)\sin(\phi_2) = 0 \quad (3-12)$$

Since $\phi_{2i-1} = \Omega\eta_i \varepsilon_{2i-1}$ and $\phi_{2i} = \Omega\eta_i \varepsilon_{2i}$ ($i=1, 2$), it gives

$$\phi_1 = \Omega\eta_1 \varepsilon_1 = \Omega\eta_1 \varepsilon$$

$$\phi_2 = \Omega\eta_1 \varepsilon_2 = \Omega\eta_1 (1 - \varepsilon)$$

$$\phi_3 = \Omega\eta_2 \varepsilon_3 = \Omega\eta_2 [1/2 + \lambda_2(\varepsilon - 1/2)]$$

$$\phi_4 = \Omega\eta_2 \varepsilon_4 = \Omega\eta_2 [1/2 - \lambda_2(\varepsilon - 1/2)]$$

Thus, Eq. (3-12) can be rewritten as

$$\begin{aligned} & \gamma_1 \sin(\Omega\eta_1) \sin \left\{ \Omega\eta_2 \left[\frac{1}{2} + \lambda_2 \left(\varepsilon - \frac{1}{2} \right) \right] \right\} \sin \left\{ \Omega\eta_2 \left[\frac{1}{2} - \lambda_2 \left(\varepsilon - \frac{1}{2} \right) \right] \right\} \\ & + \gamma_2 \sin(\Omega\eta_2) \sin(\Omega\eta_1 \varepsilon) \sin[\Omega\eta_1 (1 - \varepsilon)] = 0 \end{aligned} \quad (3-13)$$

By inspecting the above system characteristic equation, it is proposed that the frequency ratio η_i , the length ratio λ_i , the segment ratio ε and the mass-tension ratio γ_i will all affect the in-plane modal behaviour of a cable network. Refer to Eqs (9g') and (11g'), it can be seen that for a general cable network containing N main cables, the total number of main cables, N , will also influence the dynamic response of the system. In this section, the roles of all these five system parameters on the in-plane dynamic behaviour of a general cable network will be explored in detail.

3.4.1 Length ratio

To investigate the effect of length ratio, three symmetric two-cable networks, one SMT system and two DMT systems, were studied. The target cable (main cable 1) of these three networks are the same. The mass-tension ratio parameters are

Network-1 (DMT): $\gamma_1=1.0$ $\gamma_2=0.67$

Network-2 (SMT): $\gamma_1=1.0$ $\gamma_2=1.0$

Network-3 (DMT): $\gamma_1=1.0$ $\gamma_2=1.5$

By varying the length ratio, λ_2 , in all three networks from 1.0 to 2.0, their fundamental system frequencies corresponding to the segment ratio of $\varepsilon=1/4$, $1/3$ and $1/2$ are portrayed in Figures 3-8(a) to 3-8(c), respectively.

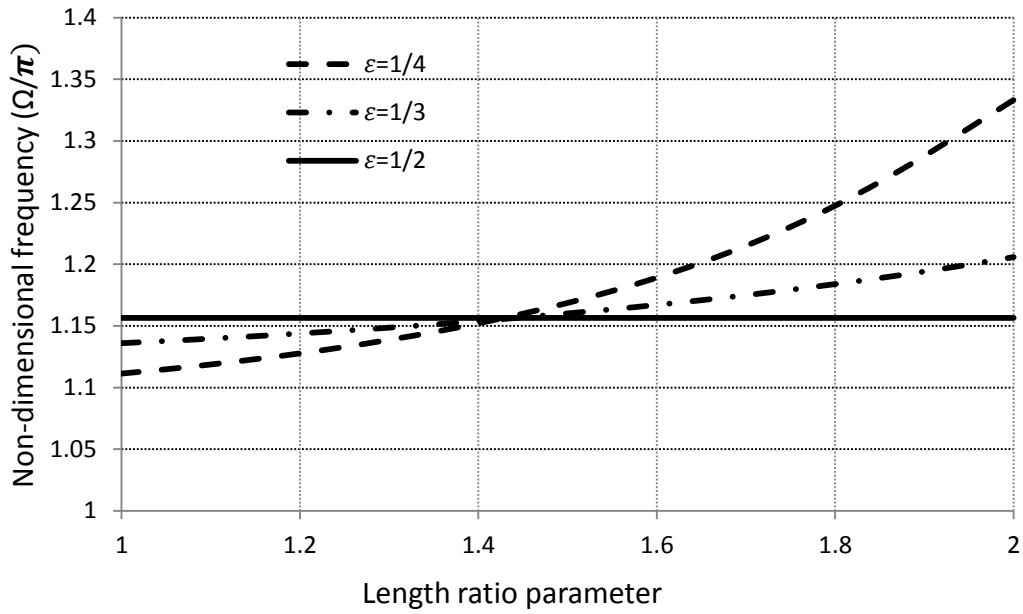


Figure 3-8(a) Non-dimensional modified frequency, Ω/π , as a function of the length ratio parameter, λ , for three different cross-tie positions (Symmetric DMT cable network, $\gamma_2=0.67$)

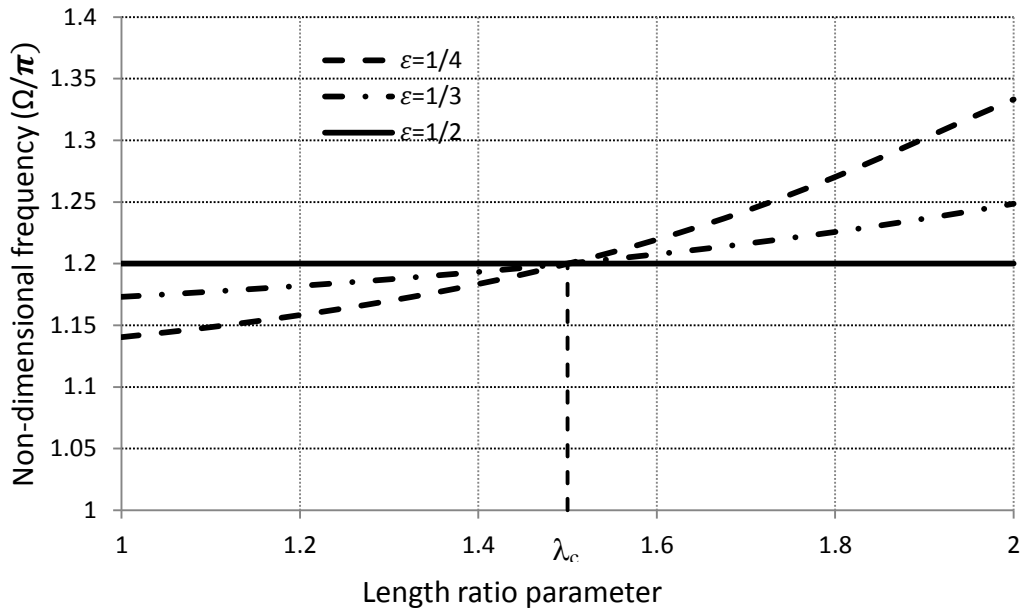


Figure 3-8(b) Non-dimensional modified frequency, Ω/π , as a function of the length ratio parameter, λ , for three different cross-tie positions (Symmetric SMT cable network, $\gamma_2=1.0$)

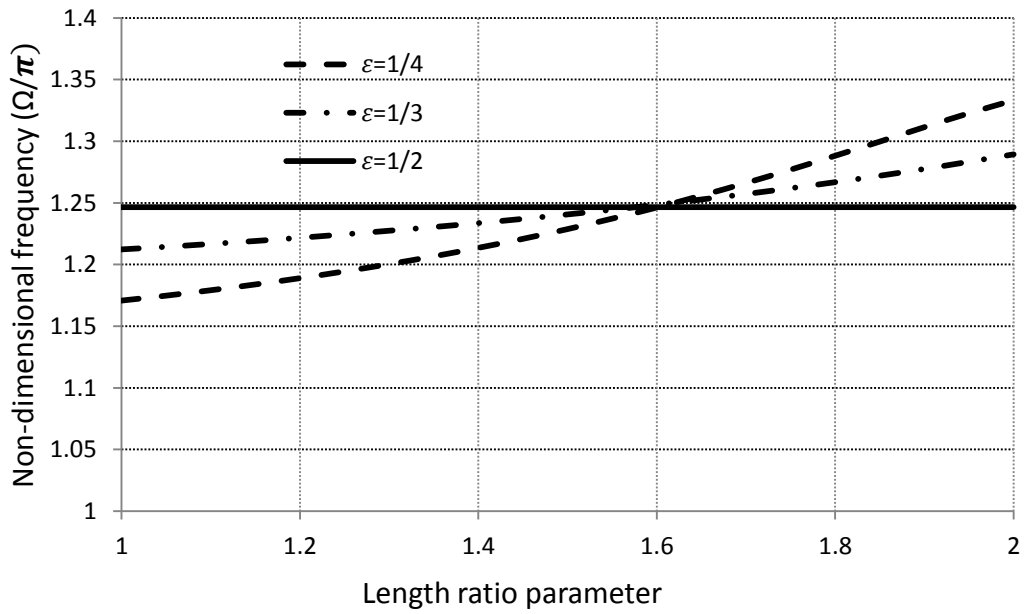


Figure 3-8(c) Non-dimensional modified frequency, Ω/π , as a function of the length ratio parameter, λ , for three different cross-tie positions (Symmetric DMT cable network, $\gamma_2=1.5$)

It can be observed in Figure 3-8 that for all three cable networks, when the rigid cross-tie is placed at the mid-span, i.e. $\varepsilon=1/2$, the fundamental frequency of the network is independent of

the length ratio λ_2 . It remains at 1.16π , 1.20π , 1.25π for networks 1 to 3, respectively. In addition, if main cable 2 in a network has higher mass-tension ratio γ_2 , the network will have a higher fundamental frequency.

For segment ratios other than $1/2$, the system fundamental frequency shows monotonic increment with respect to higher length ratio λ_2 . If the mass property and the pretension of main cable 2 remain unchanged, a higher length ratio λ_2 physically represents a more stiff cable. This implies that by connecting the target cable (main cable 1) to a stiffer neighbouring cable (main cable 2) would enhance the in-plane stiffness of the resulting cable network, which agrees with the findings in the companion paper (Ahmad and Cheng, 2012). Comparisons of the Ω - λ_2 relation curves corresponding to the three studied segment ratios show that if the cross-tie locates further away from the mid-span, i.e. varies from $\varepsilon=1/2$ to $1/4$, the system fundamental frequency becomes more and more sensitive to the length ratio between the target and the neighbouring cable. For example, in the case of Network-2, which is a SMT network, it can be seen in Figure 3-8(b) that the system fundamental frequency corresponding to the two extreme length ratio values $\lambda_2=1.0$ and $\lambda_2=2.0$ are 1.17π and 1.25π respectively when $\varepsilon=1/3$, whereas they are 1.14π and 1.33π respectively when $\varepsilon=1/4$, i.e. increased by 6.4% in the former and 17% in the latter. The same phenomenon can also be observed from Figures 3-8(a) and 3-8(c), of which both are DMT networks. For Network-1($\gamma_2=0.67$), by placing the cross-tie at $1/3$ or $1/4$ of the span, a variation of length ratio λ_2 from 1.0 to 2.0 will cause 6.1% and 20% increment in the system fundamental frequency, whereas it is 6.3% or 13.9% in the case of Network-3 ($\gamma_2=1.5$).

The most interesting finding from Figure 3-8 is that in the case of a SMT network (Figure 3-8(b)), a common intersection point for the three Ω - λ_2 curves associated with different segment ratios can be identified. A closer inspection of this point reveals that once the system parameters

η_2 and λ_2 of a symmetric SMT two-cable network satisfies the condition of $\eta_2 \cdot \lambda_2 = 1$, the change in the rigid cross-tie position will not affect the system fundamental frequency. The magnitude of the length ratio λ_2 corresponding to this intersection point is defined as the critical length ratio λ_c , of which the fundamental frequency of a symmetric SMT two-cable network will be independent of the cross-tie position. In the sub-critical length ratio range ($\lambda_2 < \lambda_c$ or $\eta_2 \cdot \lambda_2 < 1$), a higher fundamental system frequency can be achieved by moving the rigid cross-tie closer to the mid-span. However, if $\lambda_2 > \lambda_c$ or $\eta_2 \cdot \lambda_2 > 1$, i.e. in the super-critical length ratio range, the rigid cross-tie should be moved further away from the mid-span in order to achieve a higher frequency. In the case of the two DMT networks shown in Figures 3-8(a) and 3-8(c), no common intersection point of the three Ω - λ_2 curves exists, but rather, each pair of two curves intersect at a different value of length ratio λ_2 . For example, in Network-1 (Figure 3-8(a)), if $\lambda_2 = 1.42$, then placing the cross-tie at 1/3 and 1/4 span will yield the same fundamental frequency of the system. However, λ_2 should be 1.44 for $\varepsilon = 1/2$ and $\varepsilon = 1/3$ to have the same frequency, and 1.43 for $\varepsilon = 1/2$ or $\varepsilon = 1/4$ to have the same frequency.

3.4.2 Segment ratio

As noticed in the previous section, the change in the cross-tie position will affect the fundamental frequency of a cable network. The effect of segment ratio (or cross-tie position) on the modal behaviour of a cable network will be further explored in this section. A total of nine symmetric two-cable networks are selected. They are divided into three groups, with each group contains three networks. Group 1 includes three symmetric SMT two-cable networks, with the same geometric layout as those in Figure 3-3. The system parameters of these networks are

Network-1:	$\eta_1 = 1.0$	$\lambda_1 = 1.0$	$\eta_1 \lambda_1 = 1.0$
	$\eta_2 = 0.67$	$\lambda_2 = 1.0$	$\eta_2 \lambda_2 = 0.67$

Network-2:	$\eta_1=1.0$	$\lambda_1=1.0$	$\eta_1\lambda_1=1.0$
	$\eta_2=0.67$	$\lambda_2=1.5$	$\eta_2\lambda_2=1.0$
Network-3:	$\eta_1=1.0$	$\lambda_1=1.0$	$\eta_1\lambda_1=1.0$
	$\eta_2=0.67$	$\lambda_2=1.8$	$\eta_2\lambda_2=1.2$

The segment ratio of these three networks varies from $\varepsilon=1/4$ to $\varepsilon=3/4$. Groups 2 and 3 contain three symmetric DMT two-cable networks each. The only difference between these networks and those in Group 1 is that all three DMT networks in Group 2 have the mass-tension ratio parameters of $\gamma_1=1$ and $\gamma_2=0.67$; whereas those in Group 3 have the mass-tension ratio parameters of $\gamma_1=1$ and $\gamma_2=1.5$.

The relation between the system fundamental frequency and the segment ratio for the three cable networks in Groups 1, 2 and 3 are plotted in Figures 3-9(a) to 3-9(c), respectively. As can be seen in Figure 3-9(a), based on the magnitude of $\eta_2\lambda_2$, the $\Omega - \varepsilon$ curves of the three networks have totally different patterns. In the case of a symmetric SMT two-cable network, if it satisfies the condition of $\eta_2\lambda_2=1.0$ (Network-2), the fundamental frequency of the system is independent of the segment ratio (cross-tie position). This agrees with the findings in the previous section. However, when $\eta_2\lambda_2<1.0$ (Network-1), the $\Omega - \varepsilon$ curve has a convex shape. This suggests that in this type of network, the cross-tie should be placed at mid-span to achieve the highest fundamental frequency of the system. On the contrary, for a network which has $\eta_2\lambda_2>1.0$ (Network-3), the $\Omega - \varepsilon$ curve has a concave shape, implying that to achieve higher fundamental frequency of this type of network, the cross-tie should be moved away from the mid-span.

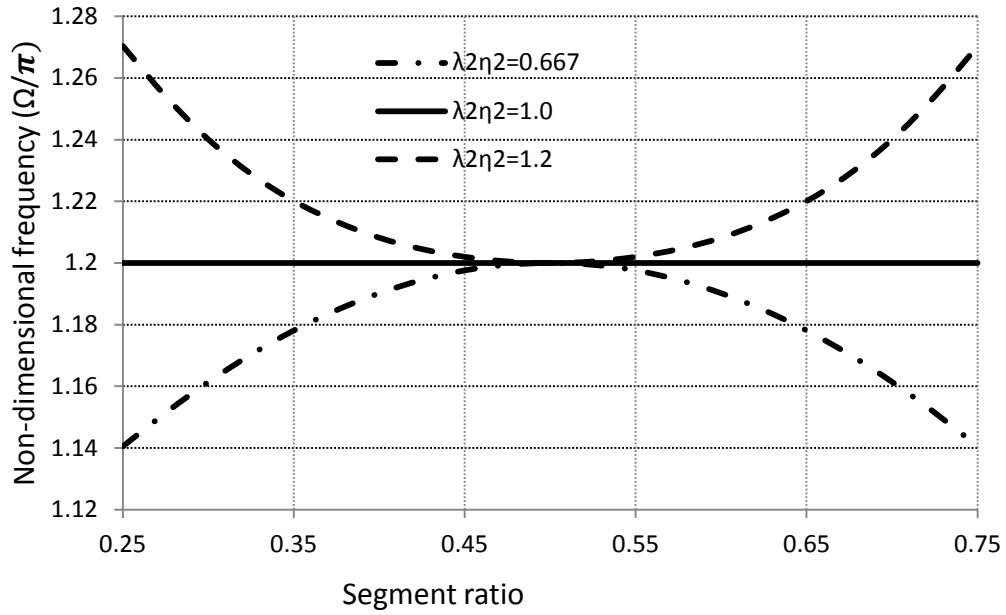


Figure 3-9(a) Non-dimensional modified frequency, Ω/π , as a function of the segment ratio parameter ε for three symmetric SMT cable networks ($\gamma_2=1.0$)

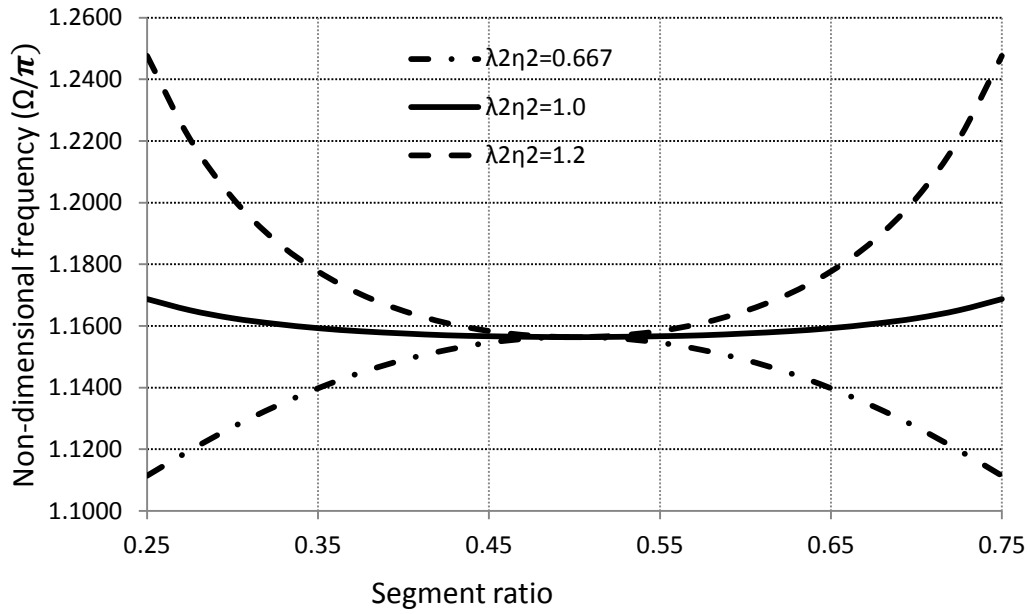


Figure 3-9(b) Non-dimensional modified frequency, Ω/π , as a function of the segment ratio parameter ε for three symmetric DMT cable networks ($\gamma_2=0.67$)

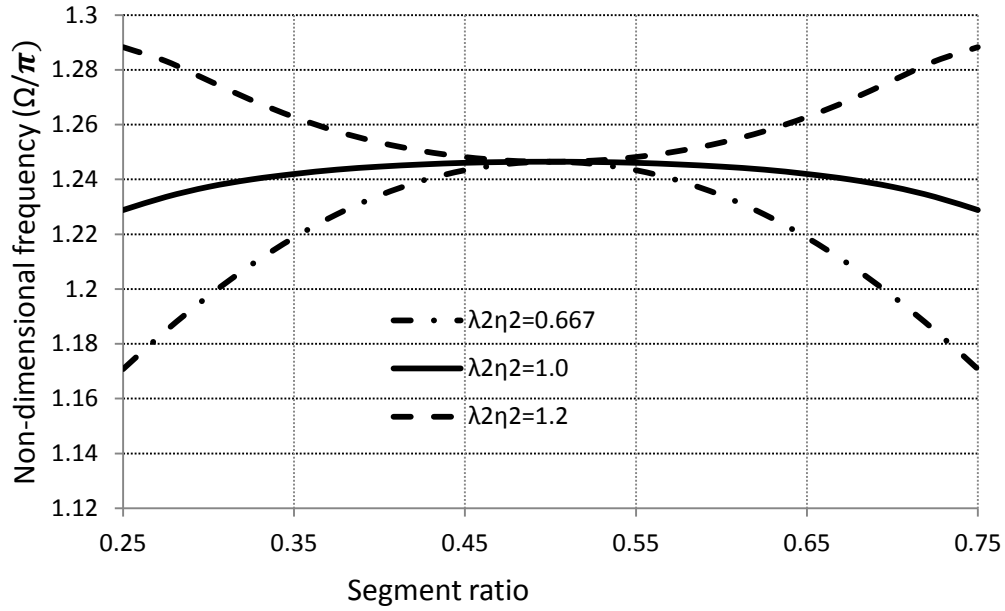


Figure 3-9(c) Non-dimensional modified frequency, Ω/π , as a function of the segment ratio parameter, ε , for three symmetric DMT cable networks ($\gamma_2=1.5$)

The same pattern of $\Omega - \varepsilon$ curves can be observed in Figures 3-9(b) and 3-9(c) for the DMT networks in Groups 2 and 3, except the system frequency is also a function of the segment ratio when the condition of $\eta_2\lambda_2 = 1.0$ (Network-2) is satisfied. In a symmetric DMT two-cable network, if $\gamma_2 < 1.0$ (Group 2), when it satisfies $\eta_2\lambda_2 \geq 1.0$, the $\Omega - \varepsilon$ curve has a concave shape, with the system frequency more sensitive to the segment ratio if $\eta_2.\lambda_2$ is larger. If $\eta_2\lambda_2 < 1.0$, convex shaped $\Omega - \varepsilon$ curve can be seen, of which by placing the cross-tie at the mid-span would yield the highest system fundamental frequency. In the case of Group 3, of which $\gamma_2 = 1.5 > 1.0$, it can be observed from Figure 3-9(c) that if the product of frequency ratio and length ratio of main cable 2 is not larger than 1, i.e. $\eta_2\lambda_2 \leq 1.0$, the cross-tie should be placed at mid-span; whereas when $\eta_2\lambda_2 > 1$, a better strategy to improve the system frequency is to place the cross-tie away from the mid-span.

3.4.3 Frequency ratio

To isolate the effect of the frequency ratio of the neighbouring cable(s) on the dynamic behaviour of the target cable (main cable 1 in the proposed analytical model in Figure 3-2) and the entire cable network, the discussion in this section will be focused on a general N -cable network having the following properties:

Frequency ratio: $\eta_1 \neq \eta_2 = \eta_3 = \dots \eta_N$

Mass-tension ratio: $\gamma_1 = \gamma_2 = \gamma_3 = \dots \gamma_N = 1$

Length ratio: $\lambda_1 \neq \lambda_2 = \lambda_3 = \dots \lambda_N$

As a first step, it was assumed that the cross-tie locates at mid-span such that the segment ratio parameters are $\varepsilon_j = 1/2 (j = 1 \text{ to } 2N)$. Substitute these non-dimensional system parameters into Eq. (3-5), and represent the frequency ratio, mass-tension ratio and length ratio of cables 3 to N using η_2 , γ_2 and λ_2 , respectively. After trigonometric simplification, yields

$$\sin\left(\frac{\Omega}{2}\eta_1\right)\left[\sin\left(\frac{\Omega}{2}\eta_2\right)\right]^{2(N-1)-1}\left[\sin\frac{\Omega}{2}(1+\eta_2)+(N-2)\sin\left(\frac{\Omega}{2}\right)\cos\left(\frac{\Omega}{2}\eta_2\right)\right]=0$$

As discussed earlier, by setting the two Sine terms outside the square brackets respectively to be zero will give a total of $(2N-2)$ sets of local modes, with some of them dominated by a single main cable, and some contains vibration coupling of more than one cable. In particular, the condition of $\left[\sin\left(\frac{\Omega}{2}\eta_2\right)\right]^{2(N-1)-1}$ will generate $(2N-3)$ local modes possessing the same modal frequency, but different mode shapes. The modal properties of the network global modes can be determined by setting

$$\sin\frac{\Omega}{2}(1+\eta_2)+(N-2)\sin\left(\frac{\Omega}{2}\right)\cos\left(\frac{\Omega}{2}\eta_2\right)=0 \quad (3-14)$$

It can be observed from Eq. (3-14) that if all the neighbouring cables have the same frequency ratio, the same mass-tension ratio and the same length ratio, the modal frequencies of

such a cable network will be dictated by the frequency ratio η_2 of the neighbouring cables and the total number N of the main cables in the network. The effect of these two parameters i.e., frequency ratio η and total number of main cables N , on the fundamental frequency of four symmetric SMT cable networks ($N=2, 3, 4, 5$) is shown in Figure 3-10(a). It can be seen that in order to increase the fundamental frequency of such type of cable network, the target cable should be connected to neighbouring cable(s) of lower frequency ratio. If the connected neighbouring cable(s) has/have the same frequency ratio as the target cable, the fundamental frequency of the cable network remains the same as that of the single target cable (main cable 1) and is irrelevant to the total number of connected neighbouring cables. This is reflected by the rightmost point in the figure, of which as the frequency ratio approaches to 1, the fundamental frequencies of the four cable networks, with the total number of cables varying from 2 to 5, all converge to the fundamental frequency of the target cable. On the other hand, the leftmost point of the figure implies that when the frequency ratio approaches to zero, i.e. the target cable is connected to extremely rigid neighbouring cable(s), its fundamental frequency can be doubled. This phenomenon is also independent of the number of rigid neighbouring cables being connected. Observations from these two extreme cases suggest that when the neighbouring cables are all rigid or all have the same frequency ratio as that of the target cable, including more cables in the network will not help to further increase the fundamental frequency of the system. In between these two extreme cases, results show that connecting a target cable with more neighbouring cables would be beneficial for enhancing its in-plane stiffness.

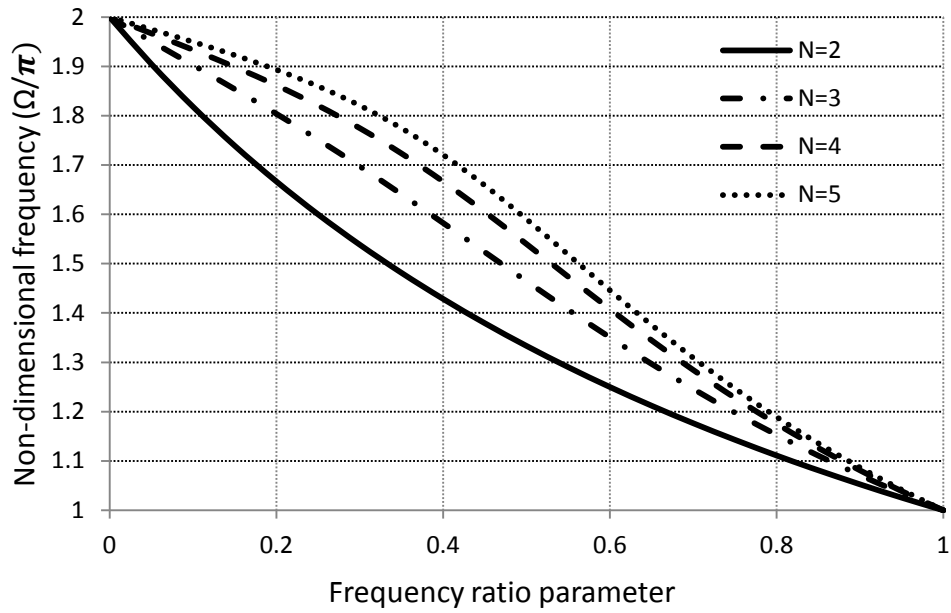


Figure 3-10(a) Non-dimensional modified frequency, Ω/π , as a function of the frequency ratio parameter, η , for four symmetric SMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at mid-span ($\epsilon=1/2$)

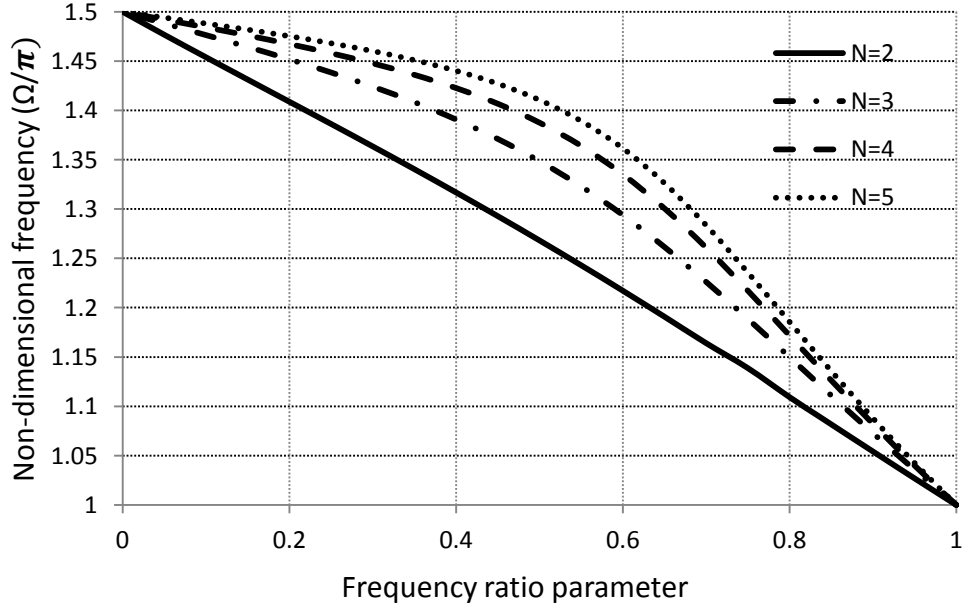


Figure 3-10(b) Non-dimensional modified frequency, Ω/π , as a function of the frequency ratio parameter, η , for four symmetric SMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at $1/3$ span ($\epsilon=1/3$)

For example, if all the neighbouring cable(s) has/have a frequency ratio of 0.6, by connecting the target cable to one, two, three or four of such neighbouring cable(s), its fundamental frequency is found to be increased by 25%, 35%, 41% and 45%, respectively.

Secondly, the cross-tie position was chosen to be at 1/3 span length from the left end of the target cable. The effect of frequency ratio on the system frequency is depicted in Figure 3-10(b). Similar phenomenon as discussed earlier for Figure 3-10(a) can also be observed from Figure 3-10(b).

In addition, by comparing the patterns of the $\Omega - \eta$ curves in Figures 3-10(a) and 3-10(b), it is found that by placing the cross-tie(s) further away from the cable mid-span, the fundamental frequency of the cable network becomes less sensitive to the frequency ratio of the neighbouring cables. If take the five-cable network ($N=5$) as an example, by changing the frequency ratio η_i ($i=1$ to 5) from 0.2 to 0.6, with the cross-ties at mid-span, the non-dimensional fundamental frequency of the network is reduced from 1.9π to 1.45π by 24%, whereas if the cross-tie(s) are moved to 1/3 span, the non-dimensional fundamental frequency is decreased by 8% from 1.48π to 1.36π .

3.4.4 Mass-tension ratio

In Section 3.3.3, it was observed that the fundamental frequency of a DMT cable network is slightly different from that of a corresponding SMT cable network, and the difference depends on the mass-tension ratio parameter.

To investigate how mass-tension ratio parameter would affect the modal behaviour of a cable network, four symmetric DMT cable networks are studied in this section, with the number of main cables in each network varies from 2 to 5. To better reveal the role of this system parameter, the frequency ratio and the length ratio of all the cables in each network are taken as

Frequency ratio: $\eta_1 \neq \eta_2 = \eta_3 = \cdots \eta_N$ $(N=2, 3, 4, 5)$

Length ratio: $\lambda_1 \neq \lambda_2 = \lambda_3 = \cdots \lambda_N$ $(N=2, 3, 4, 5)$

Mass-tension ratio: $\gamma_1 \neq \gamma_2 = \gamma_3 = \cdots \gamma_N = 0 \sim 2$ $(N=2, 3, 4, 5)$

The impact of the mass-tension ratio parameter on the fundamental system frequency of the above four cable network are examined for two cross-tie positions of $\varepsilon=1/2$ and $\varepsilon=1/3$, with the results portrayed in Figures 3-11(a) and 3-11(b), respectively. Results show that when a rigid cross-tie is placed at mid-span (Figure 3-11(a)), the fundamental frequencies of all four studied networks increase monotonically with the increase of mass-tension ratio parameter. The more neighbouring cables are connected to the target cable (main cable 1), the more increment is found for the system frequency. This implies that by connecting the target cable to more neighbouring cables with high mass-tension ratio will be beneficial to improve the in-plane stiffness, and thus modal frequencies of a target cable. Further, a comparison between Figures 3-11(a) and 3-11(b) suggests that the relation between the system fundamental frequency and the mass-tension ratio parameter is hardly been influenced by the cross-tie position. The $\Omega - \gamma$ curves in the two figures not only have the same pattern, but also almost the same values.

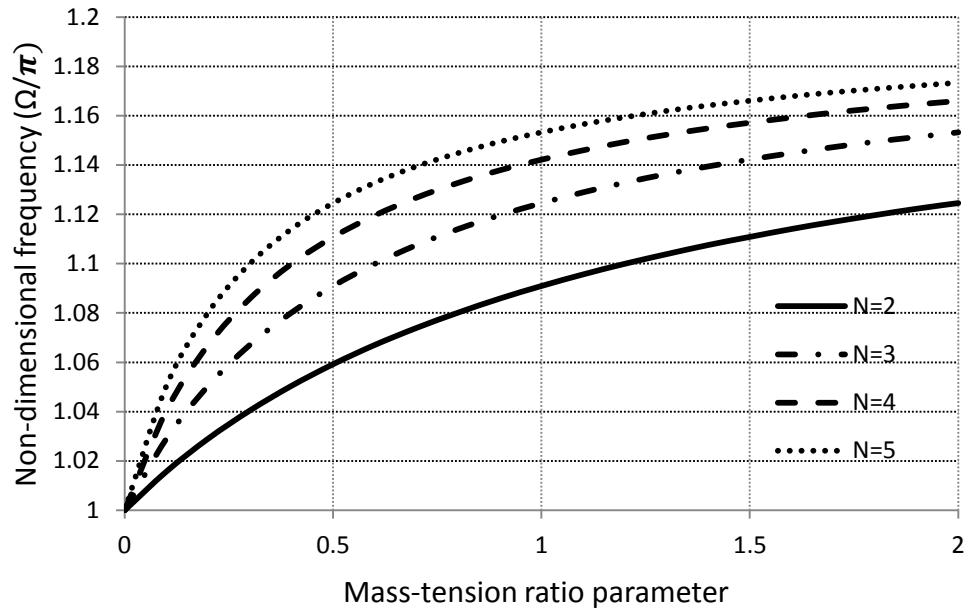


Figure 3-11(a) Non-dimensional modified frequency, Ω/π , as a function mass-tension parameter, γ , for four symmetric DMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at mid-span ($\epsilon=1/2$)

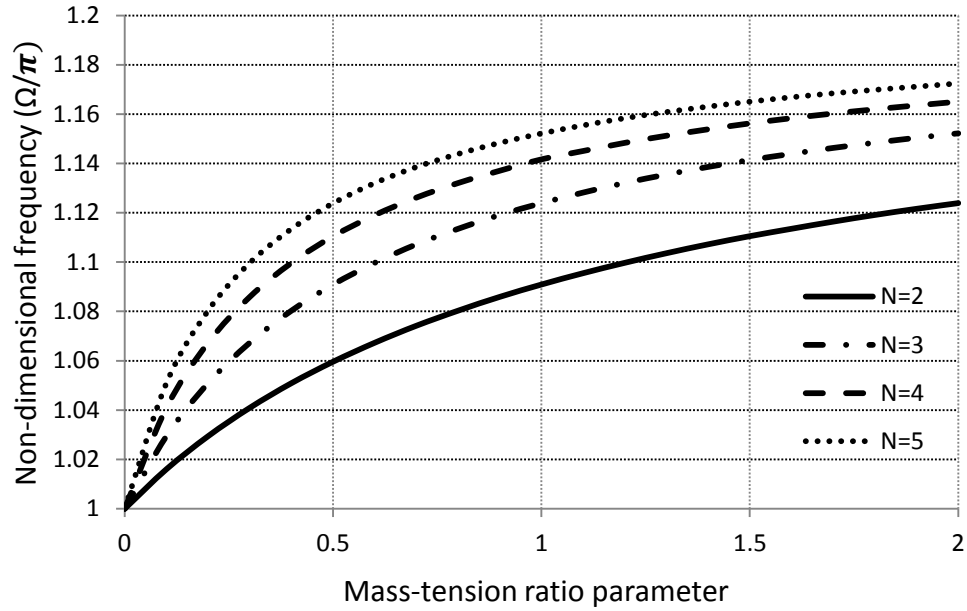


Figure 3-11(b) Non-dimensional modified frequency, Ω/π , as a function mass-tension parameter, γ , for four symmetric DMT cable networks with different number of main cables ($N=2, 3, 4, 5$) and rigid cross-tie position at 1/3 span ($\epsilon=1/3$)

3.4.5 Number of main cables

When investigating the effects of frequency ratio and mass-tension ratio on the modal behaviour of a general cable network in the previous sections, network system containing different number of main cables have been studied, with the modal analysis results presented in Figures 3-10 and 3-11. To more clearly identify how the number of connected main cables in a network system would influence its in-plane stiffness and thus in-plane vibration frequencies, Figures 3-10 and 3-11 are revisited in this section.

By comparing all the $\Omega - \eta$ curves and $\Omega - \gamma$ curves associated with different number of main cables ($N = 2$ to 5) in these two figures, it is obvious that in general by connecting the target cable (main cable 1) to more neighbouring cables, the fundamental frequency of the cable network can be improved more. This kind of increment in the system frequency is found to be more considerable when a cable network is expanding from a two-cable system to a three-cable system. However, if continuously including more neighbouring cables into the network system, its benefit on enhancing the in-plane stiffness of a cable network would gradually decrease. In a circumstance like this, connecting the target cable to fewer neighbouring cables having low frequency ratio and/or high mass-tension ratio would be a more effective solution. Take the four SMT cable networks in Figure 3-10(a) as an example. Assume the frequency ratio of all neighbouring cable(s) as 0.4, the non-dimensional fundamental frequency Ω of the networks are found to be 1.43π , 1.58π , 1.67π and 1.72π when the target cable is connected to one, two, three and four of such neighbouring cables, respectively. In other words, by expanding a two-cable network gradually into a five-cable network and connecting to one more neighbouring cable in each expansion, the fundamental frequency of the network can be increased respectively by 11.3%, 5.7% and 3.0%. Similarly, for the four symmetric DMT cable networks in Figure 3-

11(a), when the mass-tension ratio parameter of all neighbouring cable(s) is 0.5, the increment in the system fundamental frequency due to including one more neighbouring cable for a two-cable, three-cable and four-cable networks are 3.0%, 1.8% and 1.1%, respectively.

In addition, extreme cases have been identified of which the system frequency is independent of the number of main cables in the network. As been observed in Figure 3-10, when the neighbouring cable(s) is/are rigid or has/have the same frequency ratio as the target cable, connecting the target cable to more neighbouring cables will not improve the in-plane stiffness of the network system. Also, if the neighbouring cable(s) has/have very light mass which is/are negligible when compared to that of a target cable or no pretension has/have been applied, no matter how many of such neighbouring cables are connected to the target cable, it will result in no improvement of the system in-plane stiffness. This is evidenced in Figures 3-11(a) and 3-11(b) by the left most point where all four cable networks show a fundamental system frequency the same as that of the target cable.

3.5 CONCLUSIONS

Bridge stay cables are vulnerable to various types of dynamic excitations due to their flexible nature and low inherent damping. Interconnecting a problematic cable with its neighbor(s) by cross-tie(s) to increase its in-plane stiffness is one of the vibration control methods used on site. To better understand the mechanisms associated with this type of field solution, the analytical model proposed in a companion paper for studying the in-plane modal behavior of a basic cable network has been extended to a general one in the present work. The studied general cable network consists of n horizontally laid main cables connected by a single line of transverse rigid cross-ties. The modal characteristics of general cable networks having different configurations and cable properties have been investigated using the proposed model and approach. The

analytical results have been found to agree well with those obtained from independent numerical simulations. From the analytical formulation, the length ratio, the segment ratio, the frequency ratio, the mass-tension ratio, and the number of main cables being interconnected have been identified as the key system parameters affecting the modal response of a network system. The impact of each parameter on the in-plane modal properties of a general cable network has been examined. The main findings from the current study are concluded as follows:

- 1) The in-plane modal behavior of a general cable network is governed by the five identified system parameters. The networks which have different geometric layout and/or cable properties but same system parameters would yield the same modal characteristics.
- 2) The fundamental frequency of a symmetric cable network is not affected by the length ratio when a single line of transverse rigid cross-ties are placed at mid-span. However, for other cross-tie positions, the system frequency increases monotonically by connecting the target cable with shorter neighboring cable(s) which has/have larger length ratio. By moving the cross-tie(s) further away from the mid-span, the fundamental frequency of a symmetric network system becomes more sensitive to the length ratio. A critical length ratio has been identified for the symmetric SMT cable network, of which the fundamental frequency of the network would not be influenced by the cross-tie position as far as the condition of $\lambda_2\eta_2=1$ is satisfied.
- 3) Depending on the product of the length ratio and the frequency ratio $\lambda_2\eta_2$ and the mass-tension ratio γ_2 , the variation of the fundamental frequency of a symmetric cable network with respect to the segment ratio (cross-tie position) manifests either a concave or a convex shape. To achieve higher system in-plane stiffness, both $\lambda_2\eta_2$ and γ_2 should be considered when choosing the most effective cross-tie position.

- 4) Connecting a target cable to stiffer neighboring cable(s) enhances the in-plane stiffness of a cable network. If the neighboring cable(s) is/are rigid and the cross-tie(s) locate(s) at mid-span, the system fundamental frequency would double that of the target cable. However, no benefit would be gained if the neighboring cable(s) has/have the same frequency ratio as the target cable.
- 5) To achieve higher in-plane stiffness of a general cable network, the target cable should be connected with neighboring cable(s) having higher mass-tension ratio. For the two studied segment ratios of $\varepsilon = 1/2$ and $1/3$, the relation between the mass-tension ratio and the system fundamental frequency is found to be hardly affected by the cross-tie position.
- 6) Comparison between modal behaviour of a symmetric SMT cable network and a corresponding symmetric DMT system reveals that if the two networks have the same system parameters except the mass-tension ratio, then only the modal properties of the global modes would be affected, whereas those of the local modes would remain the same.
- 7) In majority of the cases, connecting a target cable with more neighboring ones would help to increase the system frequency; in particular, those with either lower frequency ratio and/or higher mass-frequency ratio. However, such a benefit gradually fades with more number of such neighboring cables connected to the target one.

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CHAPTER 4

In-plane Free Vibration of a Two-cable Network with Flexible Cross-tie

4.1 Introduction

The complex behavior of cable-stayed bridges when subjected to dynamic loads, such as wind, traffic or seismic excitation, is a long standing issue in bridge engineering. Large amplitude cable vibrations are recorded in different scenarios including the interaction between different bridge components, i.e. stay cables, bridge deck and bridge tower (e.g. Virlogeux, 1998; Macdonald and Georgakis, 2002; Caetano and Cunha, 2003; Sun et al, 2003). These can cause problems related to structural safety and user discomfort. Different solutions, such as modify cable surface (Matsumoto et al, 1992; Zhan et al, 2008), install external damper (Pacheco et al, 1993; Krenk, 2000; Tabatabai and Mehrabi, 2000; Main and Jones, 2002; Fujino and Hoang, 2008; Cheng et al, 2010) and apply cross-ties (Gimsing, 1993; Yamaguchi and Nagahawatta, 1995; Virlogeux, 1998; Caracoglia and Jones, 2005b), have been used on site with various levels of success. Though at present, most cable vibration problems on cable-stayed bridges are resolved by installing external dampers the cross-tie solution is becoming more popular. The successful examples include the Faro Bridge in Denmark, the Normandie Bridge in France, the Yobuko Bridge in Japan (Virlogeux, 1998), the Fred Hartman Bridge (Caracoglia and Jones, 2005b) and the Dames Point Bridge in USA (Kumarasena et al., 2007).

Results of the past studies show that when connecting a problematic stay cable (the target cable) with its neighbour(s) using cross-ties and forming a cable network, its in-plane stiffness would be increased (Caracoglia and Jones, 2005a, 200b; Ahmad and Cheng, 2012a, 2012b), an energy redistribution would occur between cables within the network (Ehsan and Scanlan, 1990),

and extra damping could be introduced to the target cable (Yamaguchi and Nagahawatta, 1995). Virlogeux (1998) is perhaps among the first few who seriously looked at the structural impact of the cross-tie solution on bridge stay cables. He pointed out that the usage of cross-tie could help to increase the in-plane stiffness of the connected cables by reducing their free length and forming a cable network. Two types of cross-ties, rigid (stiff) and flexible (soft), were used in an experimental study by Yamaguchi and Nagahawatta (1995). Based on their results, the modal damping increment was found to be more significant when using the flexible (soft) type cross-tie. Similar phenomenon was observed in a physical tests by Sun et al (2007), which reported that the flexible cross-ties were capable of providing more damping increment than the rigid ones. However, rigid (stiff) type of cross-tie seems to be more effective in enhancing the in-plane stiffness of the connected cables. In a case study by Caracoglia and Jones (2005b), the fundamental frequency of a cable network was found to be slightly decreased by 3%, when choosing a flexible cross-tie as oppose to a rigid one Bosch and Park (2005) conducted a series of numerical simulations to study the dynamic performance of cable networks with various configurations. It was found that although increasing the size of cross-ties would increase the in-plane stiffness and thus natural frequencies of a cable network, over-sized cross-ties would render a too rigid system and make it vulnerable to local vibrations.

From the above, it can be seen that the results in the existing studies indicate that either rigid or flexible type of cross-tie has its advantages and disadvantages in affecting the structural behaviour of the connected stay cables. Therefore, it is utmost important to clearly explain the relation between the characteristics of the connected cables and cross-tie(s) with the structural response of the resulted cable networks. In earlier works by the authors, analytical models of a basic cable network (Ahmad and Cheng, 2012a) and a general cable network (Ahmad and

Cheng, 2012a), both using transverse rigid cross-ties, were developed. The present work aims at further extending the basic cable network model with a transverse rigid cross-tie (Ahmad and Cheng, 2012a) by considering the stiffness of the cross-tie in the formulation, i.e. to propose an analytical model for studying the dynamic behaviour of two horizontally laid main cables when interconnected by a transverse flexible cross-tie. Besides investigating the in-plane modal behaviour of such a cable network system, another key element of the current study is to explore the impact of cross-tie stiffness on the dynamic performance of a basic two-cable network.

4.2 Formulation of analytical model

4.2.1 Description of the system

As portrayed in Figure 4-1, the cable network studied in the present paper comprises of two unequal length main cables, with L_1 being the length of the longer cable and L_2 being that of the shorter one. They are connected through a flexible cross-tie, which divides each main cable into two segments.

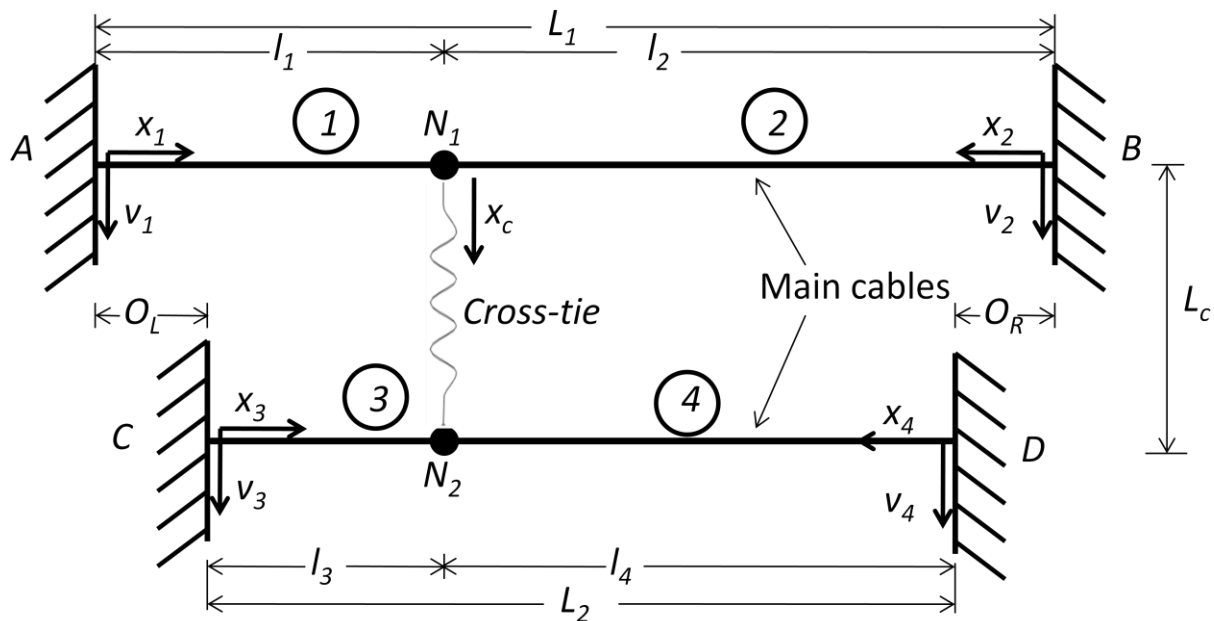


Figure 4-1 Schematic diagram of the mathematical model of a basic cable network with flexible cross-tie

The length of each cable segment and that of the transverse cross-tie are labelled as shown in Figure 4-1. Assume the mass per unit length of cable i is m_i and the pre-stressing force is H_i ($i=1, 2$). The position of the cross-tie is l_1 from the left support of main cable 1. The transverse displacements of the main cables and the axial displacement of the cross-tie are considered positive downward and negative upward. Both main cables are assumed to be fixed at both ends.

4.2.2 Derivation of the system equation

When formulating the analytical model of a two-cable network with a transverse flexible cross-tie are, the main cables are idealized as taut cables with both ends fixed. Only the in-plane transverse motions of the main cables are considered. The additional cable tension due to vibration is neglected. The flexible cross-tie is assumed to only vibrate along its axial direction and its behaviour is simulated by a linear spring connector with stiffness of K_c .

Based on the above assumptions, the in-plane free vibration of a single taut cable in its transverse direction can be expressed as (Irvine, 1974).

$$H \frac{\partial^2 v}{\partial x^2} = m \frac{\partial^2 v}{\partial t^2} \quad (4-1)$$

where H and m are the pretension and the unit mass of the cable, respectively.

The axial vibration of the flexible cross-tie can be described by the equation of motion derived by Humar (2001) for the longitudinal motion of a member with axial stiffness of EA , i.e.

$$EA \frac{\partial^2 u}{\partial x^2} = m \frac{\partial^2 u}{\partial t^2}$$

Since the axial stiffness of the cross-tie is assumed to be K_c , the above equation can be rewritten as

$$K_c \frac{\partial^2 u}{\partial x^2} = m \frac{\partial^2 u}{\partial t^2} \quad (4-2)$$

Now, by applying the Bernoulli-Fourier method to separate the variables contained in the in-plane transverse displacement $v(x, t)$ of a single main cable and the axial displacement $u(x, t)$ of the cross-tie (Ahmad and Cheng, 2012a), i.e. $v(x, t) = \bar{v}(x)\sin(\omega t)$ and $u(x, t) = \bar{u}(x)\sin(\omega t)$, the shape functions $\bar{v}(x)$ for different cable segments and $\bar{u}(x)$ for the flexible cross-tie can be expressed as

$$\bar{v}_{2i-1}(x_{2i-1}) = B_{2i-1}\sin(\Omega\eta_i x_{2i-1}/L_i) \quad i=1, 2 \quad (4-3a)$$

$$\bar{v}_{2i}(x_{2i}) = B_{2i}\sin(\Omega\eta_i x_{2i}/L_i) \quad i=1, 2 \quad (4-3b)$$

$$\bar{u}_c(x_c) = C\cos(\Omega\eta_c x_c/L_c) + D\sin(\Omega\eta_c x_c/L_c) \quad (4-3c)$$

where B_{2i-1} and B_{2i} ($i=1$ to 2) are the shape function constants of the four main cable segments, as shown in Figure 4-1; and C, D are the shape function constants of the cross-tie; $\eta_i=f_1/f_i$ and $\eta_c=f_1/f_c$ are respectively the frequency ratio of the i^{th} main cable and that of the cross-tie; f_1, f_i and f_c are the fundamental frequency of main cable 1 (the target cable), the i^{th} main cable and the cross-tie, respectively; $\Omega=\pi f/f_1$ is the non-dimensional frequency of the cable network and f is the corresponding natural frequency of the network; and L_i and L_c are the length of the i^{th} main cable and that of the cross-tie, respectively. The following boundary, compatibility and equilibrium conditions are applied to this cable network model to determine the shape function constants in Eq. (4-3):

Boundary conditions

$$\bar{v}_{2i-1}(0, t) = 0, \quad \bar{v}_{2i}(0, t) = 0 \quad i=1, 2 \quad (4-4a)$$

Compatibility conditions

$$\bar{v}_{2i-1}(l_{2i-1}, t) = \bar{v}_{2i}(l_{2i}, t) \quad i=1, 2 \quad (4-4b)$$

$$\bar{v}_3(l_3, t) - \bar{v}_1(l_1, t) = \frac{1}{K_c} [H_1(\frac{\partial \bar{v}_1}{\partial x_1} |_{x_1=l_1} + \frac{\partial \bar{v}_2}{\partial x_2} |_{x_2=L_1-l_1})] \quad (4-4c)$$

$$\bar{v}_1(l_1, t) = \bar{u}_1(0, t) \quad (4-4d)$$

$$\bar{u}_1(L_c, t) - \bar{u}_1(0, t) = \frac{1}{K_c} [H_1 (\frac{\partial \bar{v}_1}{\partial x_1} |_{x_1=l_1} + \frac{\partial \bar{v}_2}{\partial x_2} |_{x_2=L_1-l_1})] \quad (4-4e)$$

Equilibrium conditions

$$\sum_{i=1}^2 (\frac{\partial \bar{v}_{2i-1}}{\partial x_{2i-1}} |_{x_{2i-1}=l_{2i-1}} + \frac{\partial \bar{v}_{2i}}{\partial x_{2i}} |_{x_{2i}=l_{2i}}) H_i = 0 \quad (4-4f)$$

Implementing the above conditions to Eq. (4-3) and express the resulting equations into a matrix form, yields

$$[R]\{X\}=\{0\} \quad (4-5)$$

where

$$[R] = \begin{bmatrix} \sin(\phi_1) & -\sin(\phi_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & -\sin(\phi_4) & 0 & 0 \\ \psi_1 \Omega \cos(\phi_1) + \sin(\phi_1) & \psi_1 \Omega \cos(\phi_2) & -\sin(\phi_3) & 0 & 0 & 0 \\ \sin(\phi_1) & 0 & 0 & 0 & -1 & 0 \\ \psi_1 \Omega \cos(\phi_1) & \psi_1 \Omega \cos(\phi_2) & 0 & 0 & \cos(\phi_c) - 1 & \sin(\phi_c) \\ \gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4) & 0 & 0 \end{bmatrix}$$

is the coefficient matrix, $\{X\} = [B_1 \ B_2 \ B_3 \ B_4 \ C \ D]^T$ is the vector containing all the unknown shape function constants, and $\{0\}$ is the null vector. In the coefficient matrix $\{R\}$, ϕ in its general form, for the left segment of each main cable, is $\phi_{2i-1} = \Omega \eta_i \varepsilon_{2i-1}$, where $\varepsilon_{2i-1} = l_{2i-1}/L_i$ is the segment ratio of the left segment of the i^{th} main cable ($i=1, 2$); whereas for the right segment of each main cable, $\phi_{2i} = \Omega \eta_i \varepsilon_{2i}$, where $\varepsilon_{2i} = l_{2i}/L_i$ is the segment ratio of the right segment of the i^{th} main cable ($i=1, 2$); $\eta_i = f_1/f_i$ and $\gamma_i = \sqrt{H_i m_i / H_1 m_1}$ are respectively the frequency ratio and the mass-tension ratio parameter of the i^{th} main cable, f_i , m_i , H_i are respectively the fundamental frequency, the unit mass and the pretension of the i^{th} cable in the network ($i=1, 2$); $\psi_1 = H_1/(K_c L_1)$ is the non-dimensional cross-tie flexibility parameter and K_c is the axial stiffness of the cross-tie.

The infinite sets of non-trivial solution to Eq. (4-5) can be obtained by setting the determinant of the coefficient matrix [R] to zero, i.e.

$$\begin{vmatrix} \sin(\phi_1) & -\sin(\phi_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\phi_3) & -\sin(\phi_4) & 0 & 0 \\ \psi_1 \Omega \cos(\phi_1) + \sin(\phi_1) & \psi_1 \Omega \cos(\phi_2) & -\sin(\phi_3) & 0 & 0 & 0 \\ \sin(\phi_1) & 0 & 0 & 0 & -1 & 0 \\ \psi_1 \Omega \cos(\phi_1) & \psi_1 \Omega \cos(\phi_2) & 0 & 0 & \cos(\phi_c) - 1 & \sin(\phi_c) \\ \gamma_1 \cos(\phi_1) & \gamma_1 \cos(\phi_2) & \gamma_2 \cos(\phi_3) & \gamma_2 \cos(\phi_4) & 0 & 0 \end{vmatrix} = 0$$

After expanding the above equation and making all the trigonometric simplifications, the following system characteristic equation can be obtained, i.e.

$$\sin(\Omega \eta_c) [\gamma_1 \sin(\Omega \eta_1) \sin(\phi_3) \sin(\phi_4) + \gamma_2 \sin(\Omega \eta_2) \sin(\phi_1) \sin(\phi_2) + \psi_1 \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2)] = 0$$

This equation is the equation of motion describing the in-plane free vibration of a basic cable network where two horizontally suspended taut cables are connected transversely through a flexible cross-tie. The form of the above equation implies that it has two sets of solution. The first set, $\sin(\Omega \eta_c) = 0$, gives the local modes of the cross-tie. It is not the focus of the current work and therefore is dropped. The second set of solution can be obtained from

$$\gamma_1 \sin(\Omega \eta_1) \sin(\phi_3) \sin(\phi_4) + \gamma_2 \sin(\Omega \eta_2) \sin(\phi_1) \sin(\phi_2) + \psi_1 \Omega \gamma_1 \gamma_2 \sin(\Omega \eta_1) \sin(\Omega \eta_2) = 0 \quad (4-6)$$

which is the characteristic equation of the basic cable network with a flexible cross-tie shown in Figure 4-1. It can be observed from Eq. (4-6) that the left hand side of the equation is the summation of three terms. The first two terms are the same as those in the characteristic equation of a basic cable network using a rigid cross-tie (Ahmad and Cheng, 2012a). The third term, which represents the impact of cross-tie stiffness on the dynamic behaviour of the cable network, is the product of six sub-terms. They are the non-dimensional flexibility parameter ψ_1 of the cross-tie, the non-dimensional system frequency Ω , the mass-tension ratio parameters γ_1 and γ_2 of the two main cables and the Sine terms of both main cables. If the cross-tie is rigid, i.e. $K_c = \infty$, the non-dimensional flexibility parameter of the cross-tie ψ_1 would become 0. Thus, the third

term in Eq. (4-6) vanishes, and the system characteristic equation becomes the same as that of a basic cable network using a rigid cross-tie derived in an earlier work by the authors (Ahmad and Cheng, 2012a).

Equation (6) can be applied to a basic cable network having any arbitrary configurations and properties to study its in-plane modal behaviour and to evaluate how the dynamic response of a cable would be altered once it is connected to its neighbours through a flexible cross-tie.

4.3 Applications to cable networks with different configurations

In this section, the proposed analytical model of a two-cable network with a flexible cross-tie will be applied to two-cable network systems with different geometric layout and cable properties. As a model validation, a corresponding finite element model of cable network will be developed in ABAQUS 6.9. The B21 beam element will be selected to simulate the main cables, whereas the SPRING2 element is chosen to simulate the flexible cross-tie. The results obtained from the proposed analytical model will be compared with those from the numerical simulations.

4.3.1 Twin-cable network with a flexible cross-tie at arbitrary location

The two main cables in this type of network are twins, i.e. they have the same frequency ratio, segment ratio and mass-tension ratio. Since the position of the cross-tie is arbitrary, it can be assumed that the cross-tie locates at a distance l_1 from the left end of main cable 1 and $l_1 \neq L/2$ (Figure 4-2). These conditions give the frequency ratios of $\eta_1 = \eta_2 = 1$, the segment ratios of $\varepsilon_1 = \varepsilon_3 = l_1/L = \varepsilon$ and $\varepsilon_2 = \varepsilon_4 = l_2/L = 1 - \varepsilon$, and the mass-tension ratios of $\gamma_1 = \gamma_2 = 1$. Substitute these non-dimensional system parameter values into Eq. (4-6), yields

$$2\sin(\Omega)\sin\left(\Omega\frac{l_1}{L}\right)\sin\left(\Omega\frac{l_2}{L}\right) + \psi_1\Omega(\sin\Omega)^2 = 0$$

or

$$\sin(\Omega)\{2\sin(\Omega\varepsilon)\sin[\Omega(1 - \varepsilon)] + \psi_1\Omega\sin(\Omega)\} = 0 \quad (4-7)$$

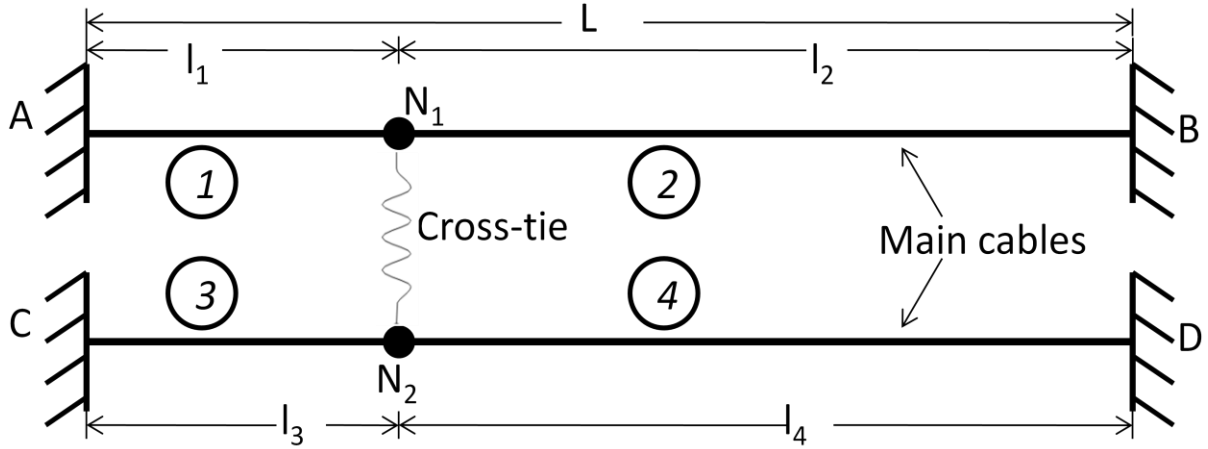


Figure 4-2 Schematic diagram of a symmetric twin-cable network with flexible cross-tie at arbitrary point

Obviously, two sets of solution are present for Eq. (4-7). Roots for the first set, yielded from $\sin(\Omega) = 0$, are responsible for the global modes of the cable network. They also exist in the twin-cable networks connected through a transverse rigid cross-tie (Ahmad and Cheng, 2012a; 2012b). This set of roots, $\Omega = n\pi$ ($n=1, 2, 3, \dots$), would give symmetric in-phase global modes for odd values of n , and asymmetric in-phase global modes for even values of n . The fundamental frequency of a twin-cable network can be obtained by setting $n=1$. It is the same as that of a single main cable in the network. This indicates that the fundamental frequency of the studied twin-cable network is independent of the type of cross-tie, be it rigid or flexible. The second set of roots, determined from $2\sin(\Omega\varepsilon)\sin[\Omega(1-\varepsilon)] + \psi_1\Omega\sin(\Omega) = 0$, are functions of the cross-tie flexibility parameter ψ_1 , and the segment ratio ε which represents the cross-tie position. These two properties of the cross-tie would dictate the modal behaviour of a twin-cable network with a flexible cross-tie. They will be further explored in the following discussion.

4.3.1.1 Special case 1: Rigid cross-tie

The axial stiffness of a flexible cross-tie is defined by the non-dimensional flexibility parameter $\psi_1 = H_1/(K_c L_1)$. Theoretically, its value varies from 0, for a rigid cross-tie, to ∞ , for

a cross-tie having no axial stiffness. However, in practice, this parameter ranges from 0.01 to 1.0 on real bridges (Caracoglia and Jones, 2005a). To verify the proposed analytical model, the special case of rigid cross-tie will be analyzed first. In the case of rigid cross-tie, its axial stiffness $K_c = \infty$, the non-dimensional flexibility parameter of the cross-tie $\psi_1 = 0$. Thus, Eq. (4-7) becomes

$$\sin(\Omega)\sin(\Omega\varepsilon)\sin[\Omega(1-\varepsilon)] = 0$$

This equation represents the system characteristic equation of a twin-cable network with rigid cross-tie and it is exactly the same as that derived in an earlier work by Ahmad and Cheng (2012a).

4.3.1.2 *Special case 2: Flexible cross-tie at quarter span*

In the cable network considered here, the flexible cross-tie is placed at quarter span of the twin main cables. Rewrite Eq. (4-7) in the form of

$$\sin(\Omega)\sin(\Omega\varepsilon)\left\{2\sin[\Omega(1-\varepsilon)] + \frac{\psi_1\Omega\sin(\Omega)}{\sin(\Omega\varepsilon)}\right\} = 0 \quad (4-8)$$

Obviously, three sets of solution are present for Eq. (4-8). The first set, yielded from $\sin(\Omega) = 0$, would produce global modes. It has been discussed earlier. The second set, which can be obtained from $\sin(\Omega\varepsilon) = 0$, generates the roots of $\Omega = n\pi/\varepsilon$. The modal frequencies obtained from this set are the same as those of the left-segment (LS) modes in a twin-cable network with rigid cross-tie determined in an earlier work (Ahmad and Cheng, 2012a). The LS mode is the mode of which the out-of-phase vibrations of the main cable left segments, i.e. cable segments 1 and 3 in Figure 4-2, take place while the right segments of the main cables (segments 2 and 4 in Figure 4-2) are at rest. The third set of roots can be obtained by setting

$$2\sin[\Omega(1-\varepsilon)] + \psi_1\Omega\sin(\Omega)/\sin(\Omega\varepsilon) = 0$$

The above expression contains two terms. The first term is the same as that in a twin-cable network with rigid cross-tie (Ahmad and Cheng, 2012a), which describes the local RS (right segment) modes. The second term appeared here because of the consideration of cross-tie flexibility. It is not only a function of the cross-tie type, but also includes the contribution of the main cable left segment motion represented by “ $\sin(\Omega\epsilon)$ ”. Therefore, compared to the rigid cross-tie case, when a flexible cross-tie is used in a twin-cable network, not only the modal frequency of the local RS modes will be changed, but more interestingly their mode shape will also be changed from the local RS mode in a rigid cross-tie network to the global mode in a corresponding flexible cross-tie network. As an example, by varying the flexibility parameter from $\psi_1=0$ (rigid cross-tie) to $\psi_1=1.0$ (flexible cross-tie), the first three local RS modes (out-of-phase) with the non-dimensional frequencies being respectively of 1.33π , 2.67π and 4π in the rigid cross-tie case are changed to the out-of-phase global modes with modal frequencies of 1.08π , 2.09π and 3.03π if cross-tie of $\psi_1=1.0$ is used. To have a more clear picture on how cross-tie flexibility would lead to such a mode shape evolution, in Figure 4-3, mode shapes of these three modes corresponding to $\psi_1=0, 0.01, 0.1$ and 1.0 are presented.

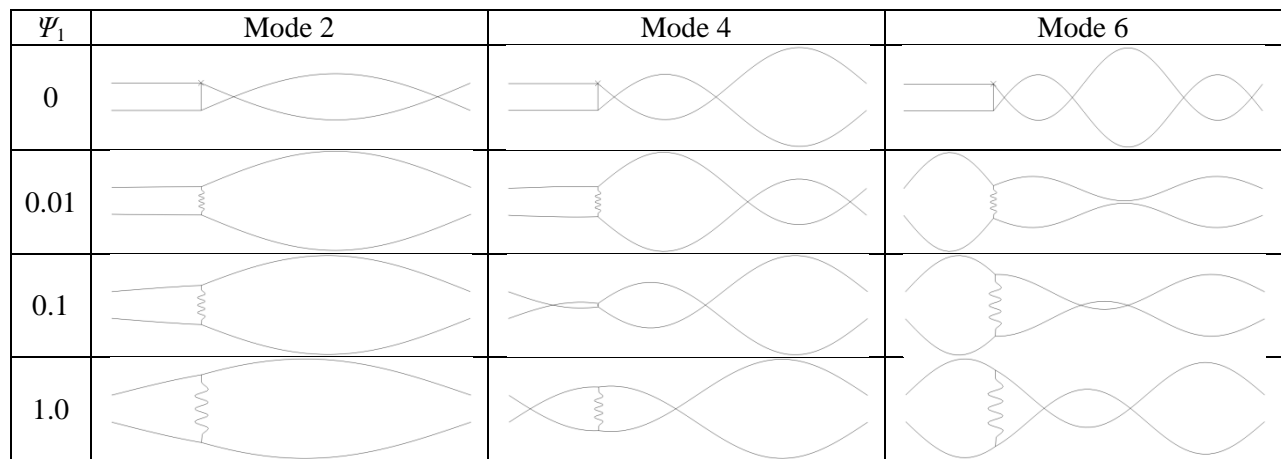


Figure 4-3 Evolution of first three right segment (RS) modes of a symmetric twin-cable network with cross-tie located at quarter span and flexibility parameter ψ_1 varies from 0 to 1.0

As can be seen from the figure, when $\psi_1=0.1$, the left segment of the main cables are started to be excited. Further reduction in the cross-tie rigidity eventually allows a full development of vibration in the cable left segments and results in a global mode. The decrease of the modal frequency of these three modes with the increase of cross-tie flexibility implies that the in-plane stiffness of a cable network will be increased the most should a rigid cross-tie is used. This analytical finding is consistent with the experimental observations by Sun et al (2007).

Numerical Example

To validate the proposed analytical model and the modal solution of a twin-cable network, a numerical example is presented in this section. The twin main cables in the example are assumed to be the same as those used in an earlier work (Ahmad and Cheng, 2012a). Both main cables have a pretension of 1598 kN, a unit mass of 47.9 kg/m, and a length of 67.34 m. The flexibility parameter of the cross-tie is assumed to be $\psi_1=1.0$ and it locates at one-fourth span from the left end of the main cables. The natural frequencies and the corresponding mode shapes of the first ten modes of this twin-cable network determined from the proposed analytical model and numerical simulations are given in Table 4-1. For comparison purpose, the modal properties of the first ten modes of the same twin-cable network but rigid cross-tie are listed in the same table.

As can be seen from the table, the modal properties of the global modes are not affected by the type of cross-tie. By replacing a rigid cross-tie with a flexible one, the modal frequency and the mode shape of both symmetric and asymmetric global modes (modes 1, 3, 5, 7 and 9 in Table 4-1 and Figure 4-3) remain the same. However, in the case of local modes, no matter dominated by vibrations of the left segments (LS modes) or the right segments (RS modes), such a change in the cross-tie type renders them to evolve into global modes, as can be seen from modes 2, 4, 6, 8, and 10 depicted in Figure 4-3. For the local LS mode (mode 8 in Table 4-1 and

Figure 4-3), only the mode shape is affected, the modal frequency of the evolved global mode in the flexible cross-tie case is still the same as the corresponding LS mode in rigid cross-tie case. Whereas for the local RS modes (mode 2, 4, 6 and 8 in Table 4-1 and Figure 4-3), both mode shapes and modal frequencies are affected. By changing cross-tie type from rigid to flexible, a local RS mode becomes a global mode with lower frequency instead. The first ten modes of the studied twin-cable network with flexible cross-tie are shown in Figure 4-4. Those of the same twin-cable network, but rigid cross-tie are also given in the figure to demonstrate the impact of cross-tie flexibility on the modal properties of a twin-cable network. In addition, the results obtained from the proposed analytical model are found to agree well with those from numerical simulations, as can be clearly seen from Table 4-1.

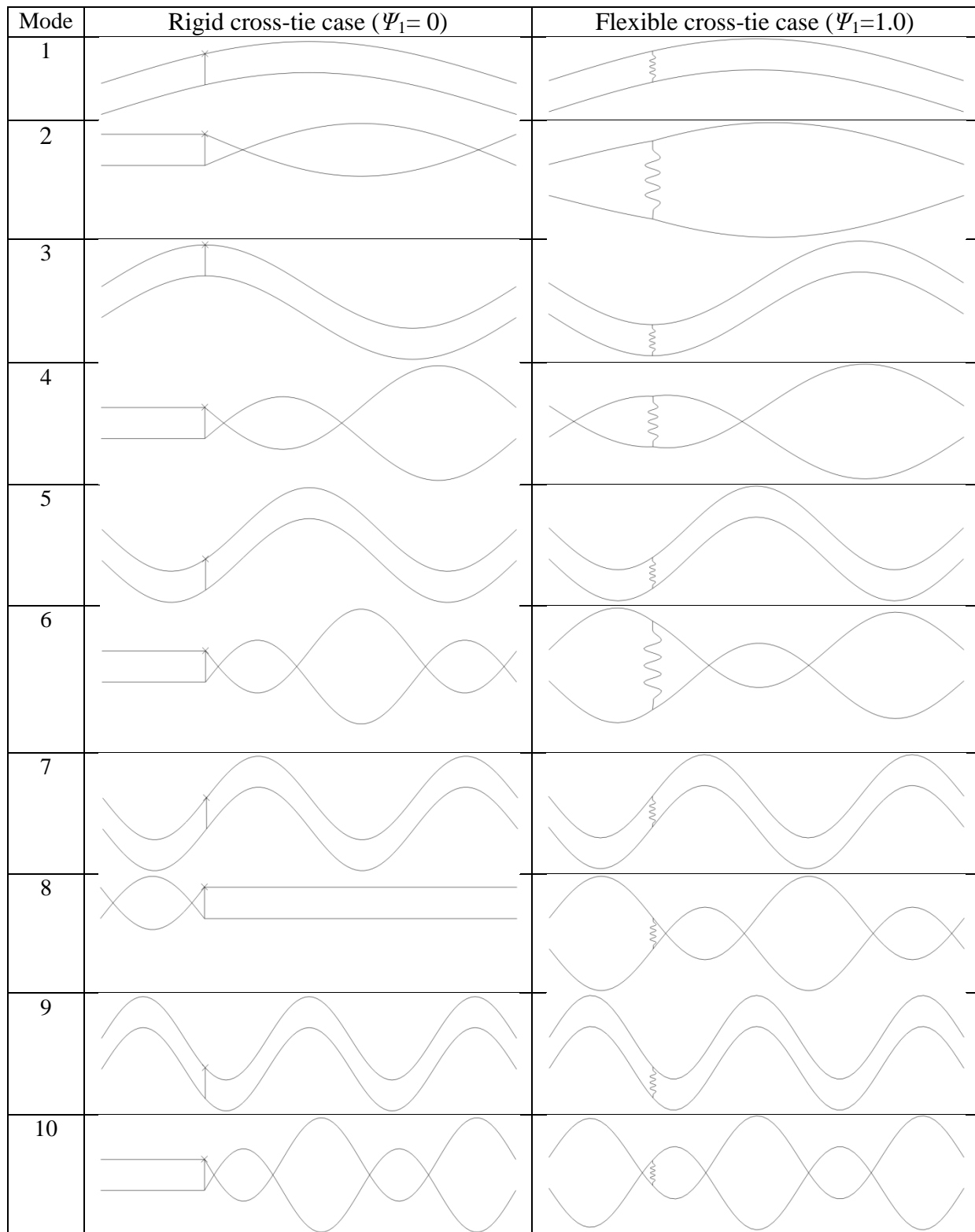


Figure 4-4 Transformation of first ten modes of a symmetric twin-cable network as flexibility parameter Ψ_1 varies from 0 to 1.0 and cross-tie locates at quarter span

Table 4-1: Comparison of the in-plane modal properties of a symmetric twin-cable network with rigid ($\psi_1=0$) and flexible cross-tie ($\psi_1=1.0$) at quarter span

Mode number	Flexible cross-tie ($\psi_1=1.0$)			Rigid cross-tie ($\psi_1=0$)	
	Modal frequency (Hz)		Mode Shape	Modal frequency (Hz)	Mode Shape
	Proposed Analytical Model	FEA			
1	1.3562	1.3562	GM, 1-Sym., in-phase	1.3562	GM, 1-Sym., in-phase
2	1.4647	1.4646	GM, out-of-phase	1.8082	LM-RS, out-of-phase
3	2.7124	2.7118	GM, 1-Asym., in-phase	2.7124	GM, 1- Asym., in-phase
4	2.8417	2.8412	GM, out-of-phase	3.6165	LM-RS, out-of-phase
5	4.0685	4.0660	GM, 2-Sym., in-phase	4.0685	GM, 2-Sym., in-phase
6	4.1161	4.1137	GM, out-of-phase	5.4247	LM-RS, out-of-phase
7	5.4247	5.4184	GM, 2-Asym., in-phase	5.4247	GM, 2-Asym., in-phase
8	5.4247	5.4184	GM, out-of-phase	5.4247	LM-LS, out-of-phase
9	6.7809	6.7682	GM, 5-Sym., in-phase	6.7809	GM, 3-Sym., in-phase
10	6.8352	6.7947	GM, out-of-phase	7.233	LM-RS, out-of-phase

4.3.2 Symmetric SMT two-cable network with flexible cross-tie at mid-span

A SMT network is a cable network of which all the main cables have the same mass-tension ratio parameter. For a two-cable network, it means $\gamma_i = 1 (i = 1, 2)$. In earlier works (Ahmad and Cheng, 2012a; 2012b), it is observed that if a target cable (main cable 1 in the proposed analytical model) is connected to a neighbouring cable having the same mass-tension ratio using rigid cross-tie, its fundamental frequency would be increased. Therefore, it will be quite interesting to see what will happen if such a cable network is equipped with flexible cross-tie. Now, consider a symmetric SMT cable network with configuration as $L_1 \neq L_2$, $H_1 = H_2$, $m_1 = m_2$ and a flexible cross-tie locates at the mid-span, as portrayed in Figure 4-5.

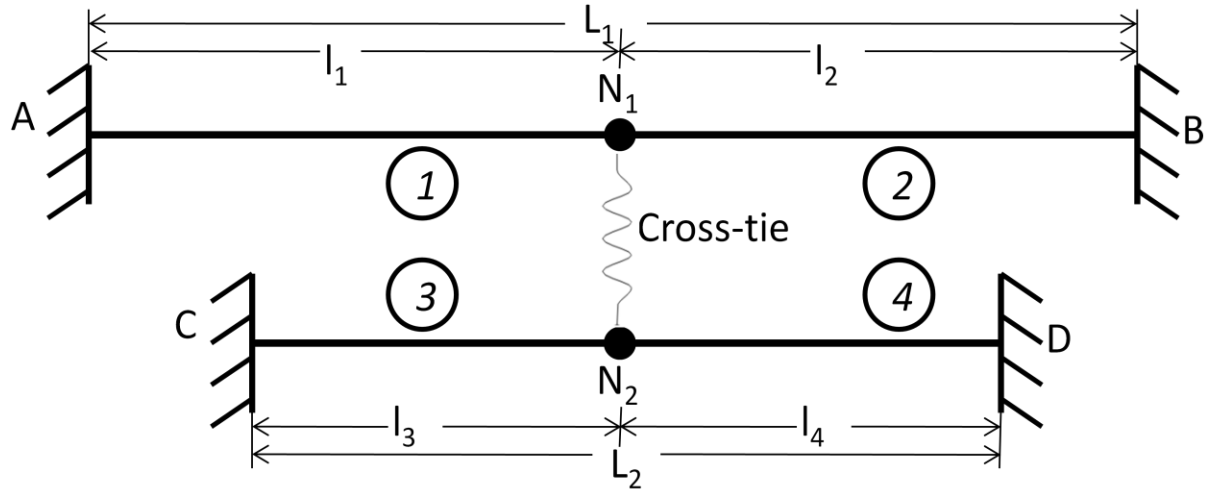


Figure 4-5 Symmetric SMT two-cable network with unequal length main cables and flexible cross-tie

The above conditions would give the frequency ratios $\eta_1 \neq \eta_2$; the segment ratios $\varepsilon_j = 1/2 (j = 1 \text{ to } 4)$; the mass-tension ratios $\gamma_1 = \gamma_2 = 1$. Also, the length of main cable 2 is chosen such that $\eta_2 \lambda_2 = 1$, where $\lambda_2 = L_1/L_2$ is the length ratio parameter of main cable 2. Substitute these non-dimensional system parameter values into Eq. (4-6), yields

$$\sin\left(\frac{\Omega}{2}\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \left\{ \sin\left[\frac{\Omega}{2}(1 + \eta_2)\right] + 2\psi_1 \Omega \cos\left(\frac{\Omega}{2}\right) \cos\left(\frac{\Omega}{2}\eta_2\right) \right\} = 0 \quad (4-9)$$

Equation (9) has three sets of solution. The first two sets are responsible for the local modes of main cables 1 and 2, respectively. These local modes were also observed in the previous works when studying a symmetric SMT two-cable network (Ahmad and Cheng, 2012a) and a symmetric SMT four-cable network (Ahmad and Cheng, 2012b) with rigid cross-tie locates at the mid-span. This suggests that in the case of a symmetric SMT cable network, local modes associated with predominant vibration of a single main cable are present for all main cables as far as the cross-tie is placed at mid-span and regardless of the cross-tie type used in the network system. The third set, derived from

$$\sin\left[\frac{\Omega}{2}(1 + \eta_2)\right] + 2\psi_1\Omega\cos\left(\frac{\Omega}{2}\right)\cos\left(\frac{\Omega}{2}\eta_2\right) = 0$$

is responsible for the global modes of the flexible cross-tie network. The first term in this condition, i.e. $\sin[\Omega(1 + \eta_2)/2]$, is exactly the same as that in the rigid cross-tie case, leaving the second term, $2\psi_1\Omega\cos(\Omega/2)\cos(\Omega\eta_2/2)$, to be responsible for the change in the modal frequency due to adoption of a flexible cross-tie. Further, the form of the second term reveals that if a flexible cross-tie is used, not only the flexibility of the cross-tie itself, but also the frequency ratio of the neighbouring cable η_2 will play a role in affecting modal frequency of the global modes.

For example, if assuming the non-dimensional flexibility parameter of the cross-tie as $\psi_1=1$ and the frequency ratio of main cable 2 as $\eta_2=0.88$, the non-dimensional fundamental frequency Ω of such a symmetric SMT two-cable network turns out to be 1.043π , whereas it is 1.06π for the rigid cross-tie case (Ahmad and Cheng, 2012a). A 1.7% reduction of the fundamental frequency occurs when a rigid cross-tie is replaced with a flexible one of $\psi_1=1$ in this case.

Numerical Example

To further validate the proposed analytical model, a corresponding numerical model of a symmetric SMT two-cable network was developed in ABAQUS 6.9-1. The physical properties of the two cables are

Main Cable 1: $H_1=1598$ kN $m_1=47.9$ kg/m $L_1=67.34$ m

Main Cable 2: $H_2=1598$ kN $m_2=47.9$ kg/m $L_2=59.52$ m

The flexibility parameter of the cross-tie is assumed to be $\psi_1=1.0$. The modal analysis results of the first ten modes determined from the proposed analytical model and numerical simulation are listed in Table 4-2, from which a good agreement between both sets can be clearly seen. Besides, for a better understanding of the impact of cross-tie type on the modal behaviour of such kind of cable network, the modal properties of the first ten modes of the flexible cross-tie network analyzed here are listed together with a corresponding rigid cross-tie system (Ahmad and Cheng, 2012a) in Table 4-3. A comparison between the modal frequencies and mode shapes of the two cable networks indicate that in the case of a symmetric SMT two-cable network, the type of cross-tie has no influence on the modal properties of the local modes dominated by motion of a single main cable. For example, the first asymmetric local mode of cable 1 (mode 2) in the rigid cross-tie case remains the same in the flexible cross-tie system, except becomes the third mode. However, it is interesting to note that the frequencies of all the global modes in the rigid cross-tie system listed in Table 4-3 are decreased when a flexible cross-tie is used. The frequency reduction is much more significant in the case of an out-of-phase global mode as compare to an in-phase one. For example, the fundamental mode of a rigid cross-tie network, which is an in-phase global mode, is reduced by 1.7% from 1.4397 Hz to 1.4149 Hz in a flexible cross-tie system; whereas mode 3 in the rigid cross-tie network, which is an out-of-phase global mode

with frequency of 2.8796 Hz, is reduced to 1.7073 Hz in the flexible cross-tie case by 41% and becomes the second mode. The mode shape of the first ten modes of this example network is presented in Figure 4-6.

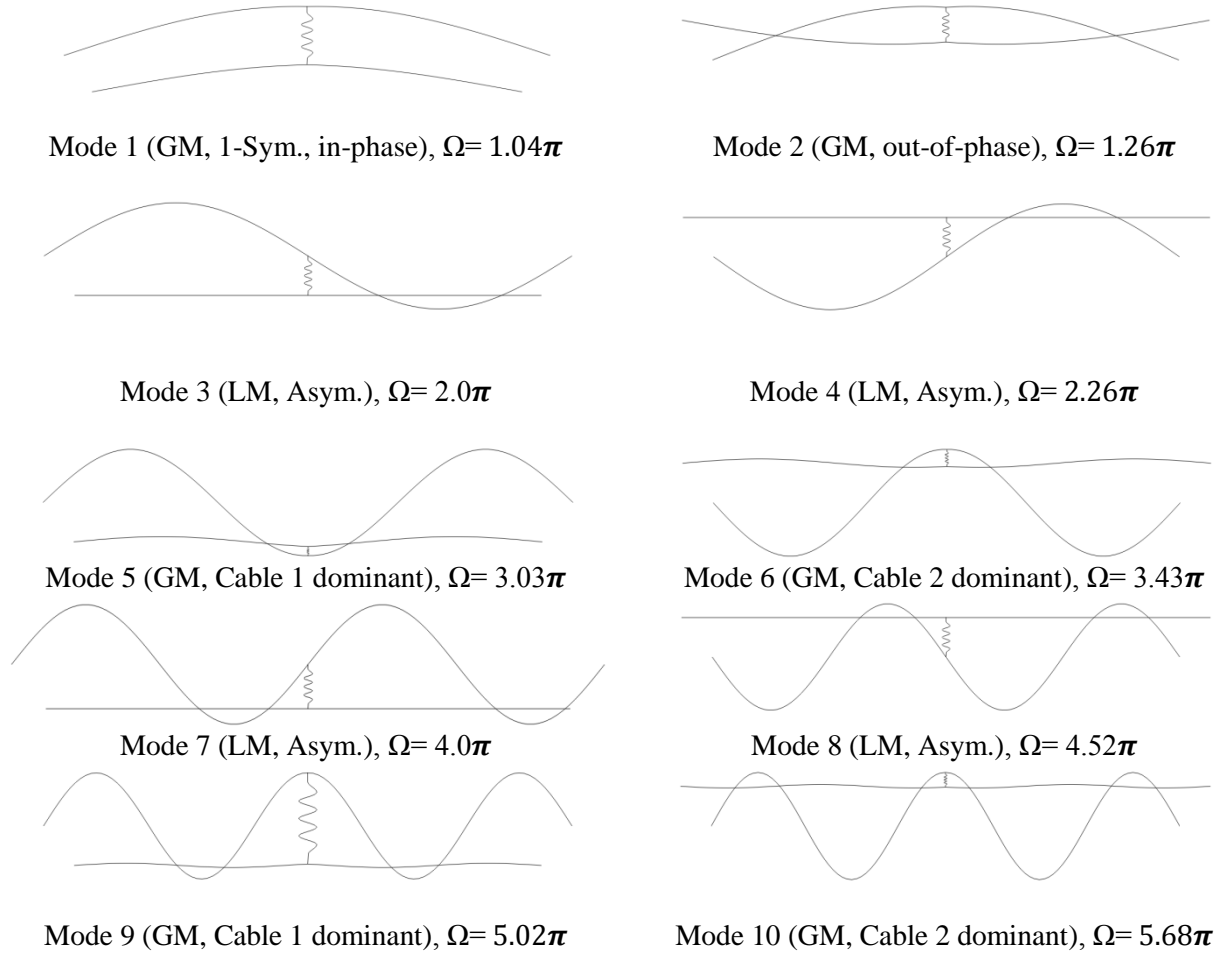


Figure 4-6 First ten modes of a symmetric SMT cable network with system parameters as frequency ratio $\eta_2=0.883$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi_1=1.0$ (GM: global mode, LM: local mode, Sym.: symmetric, Asym.: asymmetric)

Table 4-2: In-plane modal properties of a symmetric SMT two-cable network with system parameters as frequency ratio $\eta_2=0.88$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi_1=1.0$

Mode number	Modal frequency (Hz)		Mode Shape
	Proposed Analytical Model	FEA	
1	1.4149	1.4150	GM, 1-Sym., in-phase
2	1.7073	1.7075	GM, out-of-phase
3	2.7124	2.7123	LM, Cable 1, 1-Asym.
4	3.0688	3.0687	LM, Cable 2, 1-Asym.
5	4.1099	4.1090	GM, in-phase
6	4.6512	4.6503	GM, out-of-phase
7	5.4247	5.4219	LM, Cable 1, 2-Asym.
8	6.1374	6.1345	LM, Cable 2, 2-Asym.
9	6.8076	6.8016	GM, in-phase
10	7.6996	7.6932	GM, out-of-phase

4.3.3 Symmetric DMT two-cable network with a flexible cross-tie at mid-span

In this type of configuration, the two main cables have different length and their frequency ratio and mass-tension ratio parameters are both not the same. Therefore, it is termed as the DMT (different mass-tension ratio) network. Since the cross-tie is placed at the mid-span, the segment ratios of all four main cable segments are the same, i.e. $\varepsilon_j = 1/2$ ($j = 1, 2, 3, 4$). The above conditions lead to $\phi_1 = \phi_2 = \Omega/2$ and $\phi_3 = \phi_4 = \Omega\eta_2/2$. Substitute these values into Eq. (4-6), yields

$$\sin\left(\frac{\Omega}{2}\right) \sin\left(\frac{\Omega}{2}\eta_2\right) \left[\sin\left(\frac{\Omega}{2}\eta_2\right) \cos\left(\frac{\Omega}{2}\right) + \gamma_2 \sin\left(\frac{\Omega}{2}\right) \cos\left(\frac{\Omega}{2}\eta_2\right) + 2\psi_1 \Omega \cos\left(\frac{\Omega}{2}\right) \cos\left(\frac{\Omega}{2}\eta_2\right) \right] = 0 \quad (4-10)$$

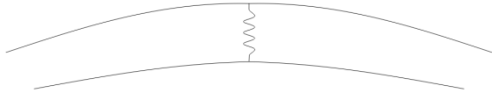
Three sets of solution are present for the above system characteristic equation, Eq. (4-10), which can be derived respectively from

$$\sin\left(\frac{\Omega}{2}\right) = 0 \quad (4-11a)$$

$$\sin\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (4-11b)$$

and
$$\sin\left(\frac{\Omega}{2}\eta_2\right)\cos\left(\frac{\Omega}{2}\right) + \gamma_2\sin\left(\frac{\Omega}{2}\right)\cos\left(\frac{\Omega}{2}\eta_2\right) + 2\psi_1\Omega\cos\left(\frac{\Omega}{2}\right)\cos\left(\frac{\Omega}{2}\eta_2\right) = 0 \quad (4-11c)$$

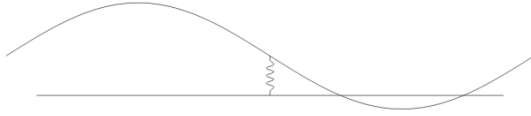
The conditions described by Eqs. (4-11a) and (4-11b) will give the local modes of main cables 1 and 2, respectively. These local modes are the same as those observed in a symmetric SMT cable network. This fact indicates that local modes dominated by a single main cable exist in both SMT and DMT networks as long as the cross-tie locates at the mid-span and the type of cross-tie, be it rigid or flexible, has no effect. As far as the global modes, given by Eq. (4-11c), are concerned, all the important system parameters, i.e. the frequency ratio η , the mass-tension ratio γ and the cross-tie flexibility ψ_1 , have their role. By taking the same network discussed in Section 3.2 but increasing the mass-tension ratio parameter of cable 2 from $\gamma_2 = 1.0$ to $\gamma_2 = 1.25$, the modal analysis results of the first ten modes are shown in Figure 4-7. By comparing with Figure 4-6, it is observed that only the modal frequencies of the global modes are slightly affected. It reveals the same fact observed in a DMT network with rigid cross-tie (Ahmad and Cheng, 2012a) that the mass-tension ratio has minor effect on the modal behaviour of a symmetric cable network. Again, a corresponding numerical simulation model was developed in ABAQUS 6.9 and the results are agreed well with the proposed analytical set. The results of the first ten modes are listed in Table 4-4.



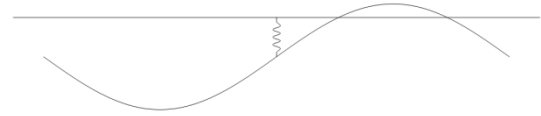
Mode 1 (GM, 1-Sym., in-phase), $\Omega = 1.047\pi$



Mode 2 (GM, out-of-phase), $\Omega = 1.24\pi$



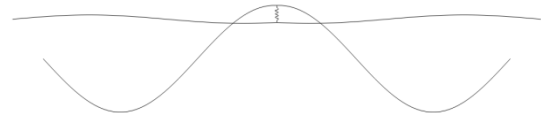
Mode 3 (LM, Asym.), $\Omega = 2.0\pi$



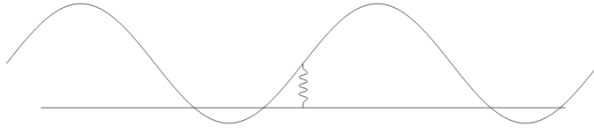
Mode 4 (LM, Asym.), $\Omega = 2.26\pi$



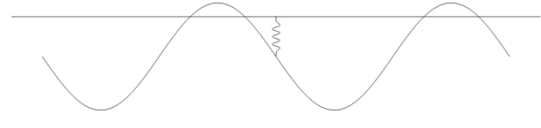
Mode 5 (GM, Cable 1 dominant), $\Omega = 3.03\pi$



Mode 6 (GM, Cable 2 dominant), $\Omega = 3.42\pi$



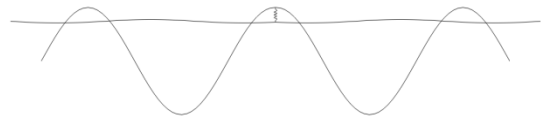
Mode 7 (LM, Asym.), $\Omega = 4.0\pi$



Mode 8 (LM, Asym.), $\Omega = 4.52\pi$



Mode 9 (GM, Cable 1 dominant), $\Omega = 5.02\pi$



Mode 10 (GM, Cable 2 dominant), $\Omega = 5.67\pi$

Figure 4-7 First ten modes of a symmetric DMT cable network with system parameters as Frequency ratio $\eta_2=0.88$, mass-tension ratio $\gamma_2=1.25$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter as $\psi_1=1.0$ (The abbreviated symbols used for describing the mode shapes are the same as those in Figure 4-6)

Table 4-3: Comparing in-plane modal properties of a symmetric SMT cable network with rigid and flexible cross-tie with system parameters as frequency ratio $\eta_2=0.88$, segment ratio $\varepsilon_j=1/2$ ($j=1$ to 4) and flexibility parameter $\psi_1=1.0$

	Flexible cross-tie		Rigid cross-tie	
	Modal frequency (Hz)	Mode Shape	Modal frequency (Hz)	Mode Shape
1	1.4149	GM, 1-Sym., in-phase	1.4397	GM, 1-Sym., in-phase
2	1.7073	GM, out-of-phase	2.7124	LM, Cable 1, 1-Asym.
3	2.7124	LM, Cable 1, 1-Asym.	2.8796	GM, out-of-phase
4	3.0688	LM, Cable 2, 1-Asym.	3.0688	LM, Cable 2, 1-Asym.
5	4.1099	GM, in-phase	4.3193	GM, in-phase
6	4.6512	GM, out-of-phase	5.4247	LM, Cable 1, 2-Asym.
7	5.4247	LM, Cable 1, 2-Asym.	5.7592	GM, out-of-phase
8	6.1374	LM, Cable 2, 2-Asym.	6.1374	LM, Cable 2, 2-Asym.
9	6.8076	GM, in-phase	7.1989	GM, in-phase
10	7.6996	GM, out-of-phase	8.1371	LM, Cable 1, 3-Asym.

4.4 Conclusion

An analytical model describing the in-plane free vibration of a basic cable network consisting of two horizontally laid taut cables interconnected by a transverse flexible cross-tie has been proposed. It has been applied to a number of two-cable networks with different configurations to study their modal behaviour. The results are found to agree well with those from numerical simulations. The impact of cross-tie stiffness on the dynamic response of a basic cable network has been discussed. The major findings of this study are concluded as follows:

- 1) The global modes of a twin-cable network remain the same when replacing the rigid cross-tie with a flexible one. The modal frequencies of the global modes are the same as those of a single cable in the network. It also implies that connecting a target cable with any neighbouring twin cable(s) would not help to increase the fundamental frequency of the network. Therefore, the global modes in a twin-cable network are independent of the cross-tie type, be it rigid or flexible.
- 2) The local modes of a twin-cable network are significantly affected by the cross-tie stiffness and evolved from a local LS or RS mode in a rigid cross-tie system to a global out-of-phase mode in a flexible cross-tie system. The flexibility of the cross-tie not only alters the mode shape but also changes the modal frequency.
- 3) For a symmetric cable network with flexible cross-tie at mid-span, it is noted that local modes dominated by vibration of a single main cables is excited. This phenomenon was also observed in the basic and the general cable networks with rigid cross-tie at mid-span. Therefore, it is reasonable to conclude that in a symmetric cable network with cross-tie at the mid-span, local modes of predominant motion of a single cable exist for all the main cables in the network.
- 4) In a symmetric cable networks with flexible cross-tie at the mid-span, the global modes are affected by the flexibility of cross-tie. If stiffer cross-tie is used, the modal frequencies would increase more.
- 5) Comparison between modal behaviour of a symmetric SMT cable network with its corresponding DMT system reveals that if the two networks have the same system parameters except the mass-tension ratio, then only the modal frequency of the global modes

would be affected, whereas the mode shapes of the global modes and all the local modes would remain the same. This is the same behaviour that was observed in rigid cross-tie case.

CHAPTER 5

Conclusions

5.1 Summary

The current study aims at studying the mechanisms and dynamic behaviour of cable networks. Three analytical models have been developed to investigate the in-plane free vibration of a basic cable network with a rigid transverse cross-tie, a general cable network with a single line of rigid transverse cross-ties, and a basic cable network with a flexible transverse cross-tie. The key system parameters of a cable network have been identified, which include the segment ratio, the frequency ratio, the mass-tension ratio, the length ratio, the cross-tie flexibility parameter, and the total number of interconnected cables. Extensive parametric study has been conducted to evaluate the role of each parameter in affecting the performance of a cable network. The main findings of the current study are concluded in the following subsections.

5.2 Basic cable network with a transverse rigid cross-tie

- 1) Connecting cables with cross-tie would induce local modes. For the twin-cable network, these local modes are dominated by vibrations of either the left or the right cable segments while the other side is at rest.
- 2) In the case of a twin-cable network, the natural frequencies and mode shapes of the global modes are the same as those of a single main cable in the network. They are independent of the position of cross-tie. However, the local modes, both right and left segment modes, purely depend on this parameter. They form complimentary pairs.
- 3) In all studied basic cable network configurations except the twin-cable network, the fundamental frequency of the cable network is higher than that of the target cable (the longer

main cable) and is a function of different system parameters. Thus, the cross-tie solution is beneficial for enhancing the in-plane stiffness of the target cable.

- 4) For a symmetric two-cable network with a rigid cross-tie at mid-span, the local modes dominated by either of the two main cables always exist and are independent of any system parameter. On the other hand, the global modes depend on both the frequency ratio and the mass-tension ratio. To further improve the in-plane stiffness of a target cable, it should be connected with neighbouring cables having lower frequency ratio and/or higher mass-tension ratio.
- 5) The global modes of a more general asymmetric two-cable network depend on the segment ratio, the frequency ratio and the mass-tension ratio. They have higher modal frequencies when compared to those of the target cable (the longer main cable).

5.3 General cable network with a single line of rigid transverse cross-ties

- 1) Five key system parameters have been identified which would dictate the in-plane modal behaviour of a general cable network connected by a single line of transverse rigid cross-ties. They are the segment ratio, the frequency ratio, the mass-tension ratio, the length ratio, and the total number of interconnected cables. Any network with different geometric layout and/or cable properties but same system parameters would yield the same modal characteristics.
- 2) In the case of symmetric networks, the local modes of all the main cables in the network always exist as far as the cross-tie(s) is/are located at the mid-span.
- 3) The fundamental frequency of a symmetric cable network is not affected by the length ratio when a single line of rigid cross-ties is placed at mid-span. However, for other cross-tie positions, the system fundamental frequency increases by connecting the target cable with

shorter neighbouring cable(s) having larger length ratio. It is also observed that the fundamental frequency of symmetric networks becomes more sensitive to the length ratio when the cross-tie is away from the mid-span. A critical length ratio has been identified for the symmetric SMT two-cable network, of which the fundamental frequency of the network would not be influenced by the cross-tie position as far as the condition of $\lambda_2 \eta_2 = 1$ is satisfied.

- 4) Depending on the product of the length ratio and the frequency ratio, i.e. $\lambda_2 \eta_2$, and the mass-tension ratio γ_2 , the variation of the fundamental frequency of a symmetric cable network with respect to the segment ratio (cross-tie position) manifests either a concave or a convex shape. To achieve higher system in-plane stiffness, cross-tie position should be close to the mid-span for $\lambda_2 \eta_2 \leq 1.0$ while away from the mid-span for $\lambda_2 \eta_2 > 1.0$.
- 5) Connecting a target cable to neighbouring cable(s) with lower frequency ratio enhances the in-plane stiffness of a cable network. If the neighbouring cable(s) is/are rigid and the cross-tie(s) locate(s) at mid-span, the system fundamental frequency would double that of the target cable.
- 6) To achieve higher in-plane stiffness of a general cable network, the target cable should be connected with neighbouring cable(s) having higher mass-tension ratio but its effect is not significant. For the two studied segment ratios of $\varepsilon = 1/2$ and $1/3$, the relation between the mass-tension ratio and the system fundamental frequency is found to be hardly affected by the cross-tie position.
- 7) Comparison between modal behaviour of a symmetric SMT cable network and a corresponding symmetric DMT system reveals that if the two networks have the same system parameters except the mass-tension ratio, then only the modal frequency of the global modes

would be affected, whereas the mode shapes of the global modes and all the local modes would remain the same.

- 8) In majority of the cases, connecting a target cable with more neighbouring ones would help to increase the fundamental frequency but this benefit is minimal when frequency ratio(s) of neighbouring cable(s) is close to 0 or 1. However, such a benefit gradually fades with more number of such neighbouring cables connected to the target one.

5.4 Basic cable network with a transverse flexible cross-tie

- 1) The global modes in a twin-cable network are independent of the type of cross-tie, be it rigid or flexible. The modal frequencies of the global modes are the same as those of a single cable in the network. Therefore, connecting a target cable with a twin neighbouring cable would not enhance its in-plane stiffness.
- 2) The local modes of a twin-cable network are significantly affected by the cross-tie type. By gradually reducing the rigidity of the cross-tie, the local modes dominated by either the left or right cable segments evolve to out-of-phase global modes. In addition, a decrease in the modal frequency occurs due to such a change in the cross-tie type.
- 3) For a symmetric cable network with cross-tie at the mid-span, local modes dominated by any of the single main cable in the network exist, despite the flexibility of the cross-tie. However, global modes in such a network configuration are affected by the type of cross-tie. Their modal frequencies are found to increase more by using stiffer type of cross-tie in the cable network.
- 4) Comparison between modal behaviour of a symmetric SMT cable network with a corresponding DMT system reveals that if the two networks have the same system parameters except the mass-tension ratio, then only the modal frequency of the global modes

would be affected, whereas their mode shapes and those of all the local modes would remain the same. The same phenomenon has been observed in rigid cross-tie case.

5.5 Suggestions for future work

An analytical model of a general cable network with a single line of transverse rigid cross-tie has been proposed in the present study. However, in real application on bridge site, multiple lines of cross-ties are a more popular scheme. A further extension of the current study should include more lines of cross-ties in the model, considering both rigid and flexible types. Effort to identify additional system parameters in such a more complicated cable network system is needed. Summarizing these analytical findings in a more design-friendly form is necessary. In addition, using cross-tie(s) to connect a target cable with its neighbour(s) would not only contribute to enhance its in-plane stiffness, but also introduce additional damping to it. Although the latter effect has been observed in a few physical tests, no analytical model is available to predict how much extra damping would incur in a particular cable network configuration. The availability of this kind of model and clarification of this extra damping issue would be much appreciated by the bridge industry.

VITA AUCTORIS

Javaid Ahmad was born in D. G. Khan, Pakistan. He obtained his B.Sc. Civil Engineering degree in 1989 from University of Engineering and Technology Lahore, Pakistan. He worked as Assistant Engineer from 1990 to 1997 in Education department of Pakistan. He earned his Post graduation diploma in Information Technology from ITI Toronto, Canada. Then, worked as Senior Software Specialist with Radix Controls Inc. Windsor, Canada from 1999 to 2009. He completed his M. Eng. (Civil) in 2009 from University of Windsor, Canada and currently a candidate for the Master's degree in Civil Engineering at the University of Windsor and hopes to graduate in winter 2012.